Chapter 10: Capital Markets and the Pricing of Risk

Fundamental question: What is the relationship between risk and return in a more complex world than the one in chapter 3 (one period, two possible outcomes)?

Key issues:

1) **Riskier investments tend to have higher average returns**
   => **compensates investors for risk**

2) **Some risk disappears in a portfolio**
   => **investors should not be compensated for this risk**
   => need to measure relationship between return and risk that does not disappear in a portfolio

10.1 Risk and Return: Insights from History

Key issues:

1) Relative risk (high to low): small firm stocks, large stocks, corporate bonds, t-bills
2) The longer the horizon, the more likely that riskier assets earn more than less risky assets

10.2 Common Measures of Risk and Return

=> need to be able to measure risk and return

A. Probability Distributions

=> to compare securities, compare returns as percent of initial investment
=> probability distribution = possible returns (R) and probability (p_R) of each possible return

Note: **need summary measures since hard to directly compare distributions**

B. Expected Return

=> **return expect to earn on average if invest in assets over and over and if the distribution does not change**
=> **the higher the number, the greater the return you can expect to earn**

$$E(R) = \sum_R p_R \times R \qquad (10.1)$$

where:
p_r = probability of return r
R = possible return

C. Variance and Standard Deviation

=> **measures how widely the possible returns are distributed**
=> **the greater the number, the wider the spread of possible returns**
=> an asset with no risk has a variance and standard deviation of zero

$$Var(R) = \sum_R p_R \times \left(R - E(R)\right)^2 \qquad (10.2)$$
$$SD(R) = \sqrt{Var(R)} \qquad (10.3)$$

volatility: standard deviation of a return

=> same units of measurement as expected return

Ex. Given the following possible returns on General Electric (GE) and General Mills (GIS) stock, calculate the expected returns and standard deviation of returns on the two stocks?

Economy	Probability	GE	GIS
Boom	.35	.38	.21
Average	.40	.15	.10
Bust	.25	–.14	.01

Expected return:

$E(R_{GE}) = .16 =$ **$.35(.38) + .4(.15) + .25(-.14)$**
$E(R_{GIS}) = .11 =$ **$.35(.21) + .4(.10) + .25(.01)$**

Standard deviation:

$Var(R_{GE}) = .35(.38 - .16)^2 + .4(.15 - .16)^2 + .25(-.14 - .16)^2 = .03926$
$StdDev\,(R_{GE}) = \sqrt{.03926} = .20$
$Var(R_{GIS}) = .35(.21 - .11)^2 + .4(.1 - .11)^2 + .25(.01 - .11)^2 = .0006273$
$StdDev\,(R_{GIS}) = \sqrt{.0006273} = .08$

=> **can expect a higher return but more uncertainty if invest in GE**

10.3 Historical Returns of Stocks and Bonds

Note: often don't know probability distributions of possible future returns on securities
=> **assume distribution of future returns will be like the past**
=> likely not the case

A. Computing Historical Returns

$$R_{t+1} = \frac{Div_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \tag{10.4}$$

Notes:

1) R_{t+1} = return actually earned between t and $t+1$ expressed as percent of what invested
2) Div_{t+1} = dividend at $t+1$
3) P_t = stock price at t
4) P_{t+1} = stock price at $t+1$
5) $\frac{Div_{t+1}}{P_t}$ =dividend yield
6) $\frac{P_{t+1} - P_t}{P_t}$ =capital gains yield
7) **must calculate a return any time a dividend is paid**

8) **can calculate at any non-dividend date by assuming a dividend of 0**

Ex. Assume the following prices and dividends for Apple (AAPL) stock

Date	Dividend	Price	Days
12/31/2020	0	132.69	
2/5/2021	0.205	136.76	36
5/7/2021	0.22	130.21	127
8/6/2021	0.22	146.14	218
11/5/2021	0.22	151.28	309
12/31/2021	0	177.57	365

What was return between 2/5 and 5/7?

$$R_{2/5\text{-}5/7} = -0.0463 = -4.63\% = \mathbf{.0016 + (-.0479)} = \frac{\mathbf{.22}}{\mathbf{136.76}} + \frac{\mathbf{130.21 - 136.76}}{\mathbf{136.76}}$$

Q: What does this tell us about Apple?

1. Calculating Realized Annual Returns

Key: usually think in terms of annual returns

a. **Assume dividends are reinvested**

$$\Rightarrow 1+R_L = (1+R_{S1})\,(1+R_{S2})\,(1+R_{S3})\ldots \qquad (10.5)$$

where:

R_L = return for longer period
Note: text only looks at determining annual returns, but can calculate for any length of time…six months, 2 years, etc.

R_{S1}, R_{S2}, etc. = returns for shorter periods
Note: text only looks at quarterly returns, but can be for any period between dividend payments…one day, two weeks, etc.

Note: compounding out returns

=> **allowing all of return to compound each period.**

Ex. Calculate the compound annual return over the year given the following returns per period (same data as previous Apple example):

Date	Dividend	Price	Return
12/31/2020	0.0000	132.690002	
2/5/2021	0.2050	136.759995	3.22%
5/7/2021	0.2200	130.210007	-4.63%
8/6/2021	0.2200	146.139999	12.40%
11/5/2021	0.2200	151.279999	3.67%
12/31/2021	0.0000	177.570007	17.38%

$1+R_{year} = 1.3465 = \mathbf{(1.0322)(0.9537)(1.124)(1.0367)(1.1738)}$

=> $R_{year} = 34.65\%$

Q: What does this tell us about Apple?

b. **Assume dividends are not reinvested**

=> solve for rate that sets PV of inflows equal to PV of outflows => NPV = 0

=> essentially solving for Internal Rate of Return (IRR)

Notes:

1) gives return on funds as long as invested in stock
   => **between time invested in stock and time cash flows thrown off as dividends or sale of stock**

2) outflows = **purchase (or beginning) price of security**
3) inflows = **dividends (or other payments), sales (or ending) price of security**

Ex. Calculate the annual return on Apple if assume dividends are not reinvested.

Date	Dividend	Price	Days
12/31/2020	0	132.69	
2/5/2021	0.205	136.76	36
5/7/2021	0.22	130.21	127
8/6/2021	0.22	146.14	218
11/5/2021	0.22	151.28	309
12/31/2021	0	177.57	365

$$NPV = -\mathbf{132.69} + \frac{\mathbf{.205}}{(\mathbf{1}+\boldsymbol{r})^{\mathbf{36/365}}} + \frac{\mathbf{.22}}{(\mathbf{1}+\boldsymbol{r})^{\mathbf{127/365}}} + \frac{\mathbf{.22}}{(\mathbf{1}+\boldsymbol{r})^{\mathbf{218/365}}} + \frac{\mathbf{.22}}{(\mathbf{1}+\boldsymbol{r})^{\mathbf{309/365}}} + \frac{\mathbf{177.57}}{(\mathbf{1}+\boldsymbol{r})^{\mathbf{365/365}}} = \mathbf{0}$$

=> Using Excel: r = .3459 = 34.59%

Q: What does this tell us about Apple?

2. Comparing Realized Annual Returns

=> can use annual returns to see which stock earned a higher return in a given year
=> given volatility of stock returns, the return for any one particular year is probably not that informative

B. Average Annual Returns

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t \qquad (10.6)$$

where:
T = number of historical returns
R_t = return over year t

=> difficult to get your mind wrapped around a list of returns => need to summarize data
=> $\bar{R}$ equals the average past return

=> **also best estimate of expected future return if distribution does not change**

C. Variance and Volatility (Standard Deviation) of Returns:

Note: variance and volatility measure **the spread of past returns**

=> volatility (or standard deviation) is in same units as average return

$$Var(R) = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2 \quad (10.7)$$

Notes:

1) dividing by *T-1* rather than *T* gives unbiased estimator
2) if calculate variance using returns for periods that are shorter than a year, calculate annual variance by multiplying calculated variance by number of periods per year

$$\text{Volatility} = SD(R) = \sqrt{Var(R)} \quad (10.A)$$

=> **gives spread of possible returns**
=> **the higher the volatility, the more spread out the returns**

Note: If calculate volatility using returns for periods that are shorter than a year, calculate annual volatility by multiplying calculated volatility by $\sqrt{number\ of\ periods\ in\ year}$

Ex. Based on the following annual returns on Apple (AAPL) and General Mills (GIS), how did the average annual returns and volatility of Apple compare to those of General Mills?

Year	Apple	GIS
2021	35%	11%
2020	82%	14%
2019	89%	43%
2018	-5%	-32%
2017	48%	-1%
2016	12%	10%

$\bar{R}_{AAPL} = +43.6\,\% = \frac{\mathbf{1}}{\mathbf{6}}(\mathbf{35 + 82 + 89 - 5 + 48 + 12})$

$\bar{R}_{GIS} = +7.5\,\% = \frac{1}{6}(11 + 14 + 43 - 32 - 1 + 10)$

Q: What do these two numbers tell us about Apple and General Mills?

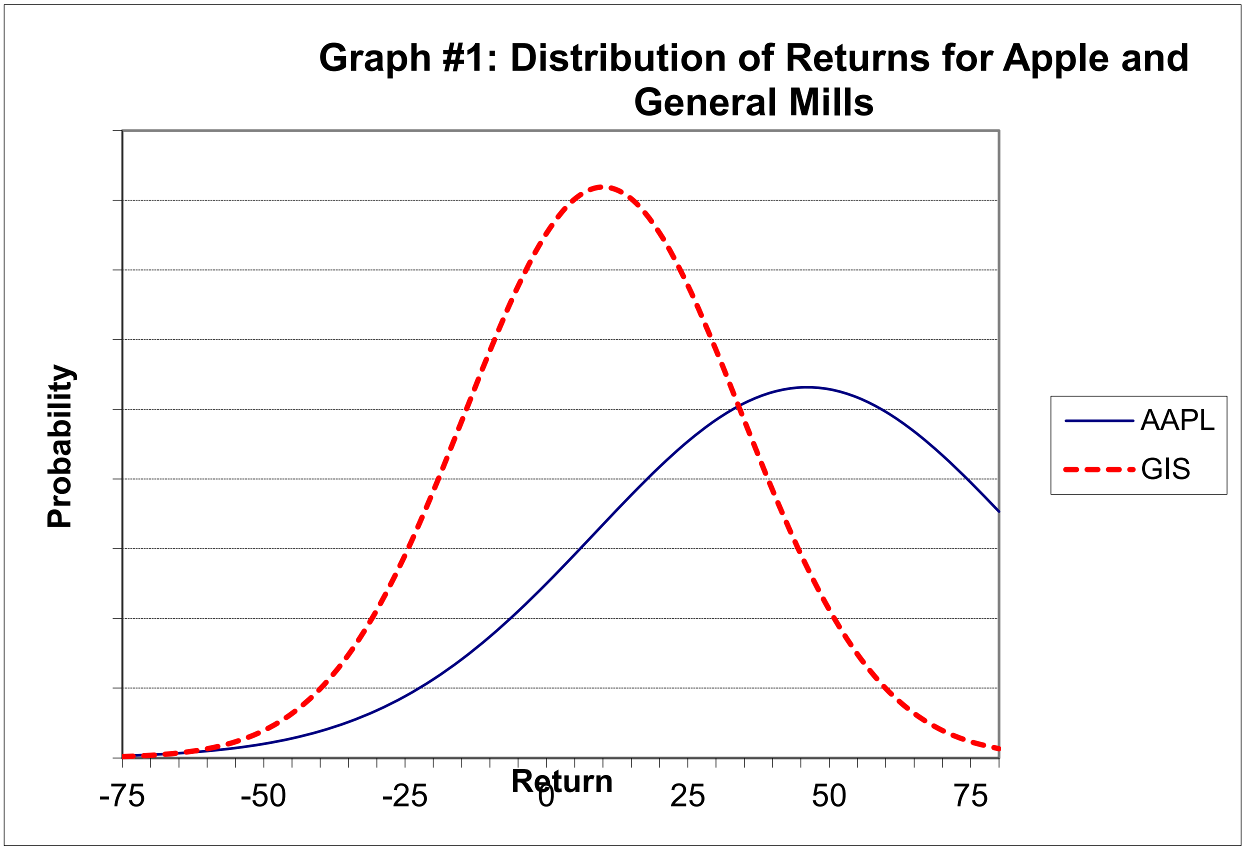
$Var(R_{AAPL}) = 1406 = \frac{1}{5}[(35 - 43.6)^2 + (82 - 43.6)^2 + (89 - 43.6)^2 + (-5 - 43.6)^2 + (48 - 43.6)^2 + (12 - 43.6)^2]$

$SD(R_{AAPL}) = 37.5\% = \sqrt{\mathbf{1406}}$

$Var(R_{GIS}) = 590.8 = \frac{1}{5}[(11 - 7.5)^2 + (14 - 7.5)^2 + (43 - 7.5)^2 + (-32 - 7.5)^2 + (-1 - 7.5)^2 + (10 - 7.5)^2]$

$SD(R_{GIS}) = 24.3\% = \sqrt{590.8}$

Q: What do the standard deviations tell us about Apple and General Mills?



Note: To create this graph I assumed that the returns of Apple and General Mills are normally distributed…which is not the case.

Q: Would you invest in Apple or General Mills? Why?

D. Standard Error (SE): Standard Deviation of Average

Notes:

1) the calculated average return is only an estimate of the true average
2) averages vary less than individual observations
3) the bigger our sample, the more confident we are that the average we calculated is the true average

=> Need some way to measure uncertainty about our estimate of the average return

1. Standard Error

Standard Error: $SE = \frac{SD}{\sqrt{N}}$ (10.8)

Where:
SE = standard error = standard deviation of average or standard deviation of returns on a portfolio of assets with independent returns
SD = standard deviation of the observations (individual returns)
N = number of observations (size of sample)

Ex. Calculate the standard error of returns on Apple in the previous example where the standard deviation of returns over six years equaled 37.5%.

SE (Average return on Apple) $= 15\% = \frac{\mathbf{37.5\%}}{\sqrt{\mathbf{6}}}$

2. Limitations of Expected Return Estimates

=> large estimation errors for average return on individual securities
=> not reliable estimate for expected return on an individual security

E. Compound Annual Return

=> return required each and every year to duplicate the return on an asset over some period

$$CAR = [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_T)]^{1/T} - 1 \quad (10.B)$$

Notes:

1) this is a geometric rather than an arithmetic average
2) the compound annual return is a better description of long-run past performance
3) the average annual return is the best estimate of an investments expected return in the future

Ex. Calculate the compound annual return on Apple and General Mills (GIS) using the data from the previous example.

Year	Apple	GIS
2021	35%	11%
2020	82%	14%
2019	89%	43%
2018	-5%	-32%
2017	48%	-1%
2016	12%	10%

$$CAR(AAPL) = .394 \ = \mathbf{[(1.35)(1.82)(1.89)(0.95)(1.48)(1.12)]^{1/6} - 1}$$

=> earning 39.4% per year every year for 6 years would provide the same return as Apple over the 6 years

$$CAR(GIS) = .05 \ = [(1.11)(1.14)(1.43)(0.68)(0.99)(1.10)]^{1/6} - 1$$

=> gaining 5% per year every year for 6 years would have provided the same return as General Mills over the 6 years

10.4 The Historical Trade-Off Between Risk and Return

A. The Returns on Large Portfolios

=> higher volatility portfolios earn higher returns
=> order (least volatile/lowest return to most volatile/highest return): T-bills, corporate bonds, S&P500, small stocks

B. The Returns of Individual Stocks

=> **no clear relationship between volatility and return**

11.1 The Expected Return on a Portfolio

Notes:

1) we'll need the material in section 11.1 of the text for the next section
2) a portfolio is defined by the percent of the portfolio invested in each asset

$$x_i = \frac{MVi}{\sum_j MV_j} \tag{11.1}$$

$$R_P = \sum_i x_i R_i \tag{11.2}$$

$$E[R_P] = \sum_i x_i E[R_i] \tag{11.3}$$

where:

x_i = percent of portfolio invested in asset i
MV_i = market value of asset i = number of shares of i outstanding × price per share of i
$\sum_j MV_j$ = total value of all securities in the portfolio
R_P = realized return on portfolio
R_i = realized return on asset i
$E[R_P]$ = expected return on portfolio
$E[R_i]$ = expected return on asset i

Ex. Assume you build a portfolio with \$10,000 invested JPMorganChase which has an expected return of 9% and \$30,000 invested in General Dynamics which has an expected return of 16%. Calculate the expected return on your portfolio.

$$x_{jpm} = .25 = \frac{\mathbf{10{,}000}}{\mathbf{10{,}000+30{,}000}}$$
$$x_{GD} = .75 = \frac{\mathbf{30{,}000}}{\mathbf{10{,}000+30{,}000}} = \mathbf{1-.25}$$

$$E(R_p) = .1425 = \mathbf{.25\times.09+.75\times.16}$$

10.5 Common Versus Independent Risk

A. Theft Versus Earthquake Insurance: An Example

B. The Role of Diversification

Ex. Assume invest 70% of your money in Honda (HMC) and 30% of your money in Lockheed Martin (LMT). How does the risk of your portfolio compare to the risk if you put everything in Honda or everything in Lockheed Martin?

	Returns on:			
Year	HMC	LMT	Porfolio	Calculation
2021	4.0%	-4.6%	1.4%	=0.7*(4)+0.3*(-4.6)
2020	2.9%	-6.5%	0.1%	=.7(2.9)+.3(-6.5)
2019	11.2%	52.5%	23.6%	=.7(11.2)+.3(52.5)
2018	-19.8%	-16.3%	-18.7%	Etc.
2017	21.0%	31.8%	24.3%	
2016	-5.8%	18.4%	1.5%	
2015	10.7%	16.2%	12.3%	
2014	-26.9%	33.8%	-8.7%	
2013	14.3%	68.1%	30.4%	
2012	24.1%	19.5%	22.8%	

Average Returns:

Note: Use equation 10.6

Honda: $3.6\% = \frac{1}{10}(4 + 2.9 + 11.2 - 19.8 + 21 - 5.8 + 10.7 - 5.8 + 10.7 - 26.9 + 14.3 + 24.1)$

Lockheed Martin: $21.3\% = \frac{1}{10}(-4.6 - 6.5 + 52.5 - 16.3 + 31.8 + 18.4 + 16.2 + 33.8 + 68.1 + 19.5)$

Portfolio: 8.9%

Note: Can calculate in two ways:

1) Use 10.6: $8.9\% = \frac{1}{10}(1.4 + 0.1 + 23.6 - 18.7 + 24.3 + 1.5 + 12.3 - 8.7 + 30.4 + 22.8)$

2) Use equation 11.3: 8.9%= **.7(3.6)+.3(21.3)**

Standard deviation of returns:

Honda: $16.7\% = \sqrt{\frac{1}{9}[(4 - 3.6)^2 + (2.9 - 3.6)^2 + \cdots + (24.1 - 3.6)^2]}$

Lockheed Martin: $24.9\% = \sqrt{\frac{1}{9}[(-4.6 - 21.3)^2 + (-6.5 - 21.3)^2 + \cdots + (19.5 - 21.3)^2]}$

Portfolio: $16.3\% = \sqrt{\frac{1}{9}[(1.4 - 8.9)^2 + (0.1 - 8.9)^2 + \cdots + (22.8 - 8.9)^2]}$

Note: portfolio is less risky than either stock by itself due to diversification

10.6 Diversification in Stock Portfolios

A. Firm-Specific Versus Systematic Risk

Note: The material in this section is one of the main ideas in finance. You will see a derivation of the math of portfolios in investments and corporate finance (FIN 4365 and 4360).

1. Firm-specific news => **creates risk called firm-specific, idiosyncratic, unique, diversifiable**

2. Market-wide news=> **creates risk called systematic, undiversifiable, market risk**

Notes:

1) Type S firms have Systematic risk and type I firms have Idiosyncratic risk.
2) Stocks differs in mix of market and company-specific risk. They also differ in how sensitive they are to market-wide news.

Ex. Kellogg is not very sensitive to how the economy is doing since people buy about the same amount of cereal regardless of how the economy is doing. But First Solar (a firm that designs, manufactures, and installs solar power systems) is highly sensitive to the overall economy since people can always delay installing solar systems.

B. No Arbitrage and the Risk Premium

Key idea: **The risk premium of a security is determined by its systematic risk and does not depend on its standard deviation.**

=> standard deviation (volatility) has no particular relationship with return since standard deviation stems in part from company-specific risk that gets diversified away in a portfolio

Comment: <u>This is one of the key ideas in finance.</u>

1. A Fallacy of Long-Run Diversification

Key issue: when diversify, spreading portfolio over a number of assets. If one asset does poorly, this does not affect the amount invested in other assets. However, if have loss in one year, it reduces the amount have to invest in all subsequent years.

10.7 Measuring Systematic Risk

A. Identifying Systematic Risk: The Market Portfolio

Efficient portfolio: **a portfolio that cannot be further diversified**

Market portfolio: **portfolio of all stocks and securities traded in capital markets**

Notes:

1) it is common practice to use the S&P500 portfolio as approximation of market
2) **I will use the S&P500 and the market portfolio interchangeably.**

B. Sensitivity to Systematic Risk: Beta

Beta: **expected % change in security's return given a 1% change in the return on the market portfolio**

1. Real-firm Betas

Note: you can look up stock betas numerous places

Ex. You can look up stock betas at Yahoo! Finance on a stock's main page.

10.8 Beta and the Cost of Capital

A. Estimating the Risk Premium

1. The Market Risk Premium

$$MRP = E[R_{Mkt}] - r_f \tag{10.10}$$

Notes:

1) **The beta of the market is 1.0**
2) **The market risk premium is the extra return that compensates investors for taking the risk of the market**
3) **I will provide either the expected return on the market (or S&P500) or the market risk premium.**

2. Adjusting for Beta

$$r_i = r_f + \beta_i \times \left(E(R_{Mkt}) - r_f\right) \tag{10.11}$$

where:

r_i = cost of capital (or required return) for asset i
r_f = risk-free rate
β_i = beta of asset i
$E(R_{mkt}) - r_f$ = market risk premium
$\beta_i \times \left(E(R_{Mkt}) - r_f\right)$ = security risk premium for asset i

Ex. Assume the risk-free rate equals 2% and that the market risk premium is 7%. What return will investors demand on Eli Lilly (LLY) which has a beta of 0.37 and on Sony (SNY) which has a beta of 1.65? What are the risk premiums on both securities?

Required returns:

r_{LLY} = .046 = **.02 + 0.37(.07)**
r_{SNY} = .136 = **.02 + 1.65(.07)**

Risk premiums:

LLY = .026 = **.37 x .07**
SNY = .116 = **1.65 x .07**

=> Sony has more risk than Eli Lilly.
=> investors demand a higher return and a greater risk premium

B. The Capital Asset Pricing Model

=> equation 10.11 is often referred to as the Capital Asset Pricing Model (CAPM)
=> most used model for estimating cost of capital used in practice

C. Risk and return in portfolios [Not in this textbook until chapter 11]

=> a portfolio is an asset like any other
=> no relationship exists between standard deviation of a portfolio and it risk premium or required return

1. risk premiums for a portfolio depends on the beta of the portfolio
=> given by equation 10.10

2. Required return on a portfolio depends on the beta of the portfolio
=> given by 10.11

3. Betas of portfolios

$$\beta_P = \sum_i x_i \beta_i \tag{11.24}$$

Ex. Assume that JPMorgan Chase (JPM) has a standard deviation of returns 20% and a beta of 1.22 and that Proctor & Gamble (PG) has a standard deviation of returns of 29% and a beta of 0.28. Assume also that the risk-free rate equals 2% and the expected return on the market equals 11%. What is beta of the portfolio where invest $300,000 in JPM and $100,000 in PG? What is the risk premium and required return on JPM, PG and the portfolio?

$x_{JPM} = .75 = \frac{\mathbf{300,000}}{\mathbf{400,000}}, x_{PG} = .25 = \frac{\mathbf{100,000}}{\mathbf{400,000}}$

=> β_P = 0.985 = **.75(1.22) + .25(0.28)**

Risk premiums:

Market = .09 = **.11 - .02**
JPM = .1098 = **1.22 x .09**
PG = .0252 = **.28 x .09**
Port = .0887 = **0.985 x .09**

Required returns:

JPM: .1298 = .02 + .1098 = **.02 + 1.22 x .09**
PG = .0452 = .02 + .0252 = **.02 + .28 x .09**
Port = .1087 = .02 + .0887 = **.02 + 0.985 x .09**

=> JPM has a higher risk premium and has a higher required return because of higher systematic risk (beta) even though it has a lower standard deviation. Much of PGs greater standard deviation disappears in a well-diversified portfolio.