Chapter 6: Valuing Bonds

Fundamental question: How we determine the value of (or return on) a bond?

6.1 Bond Cash Flows, Prices and Yields

A. Bond Terminology

$$CPN = \frac{CR \times FV}{CPY} \tag{6.1}$$

where:

CPN = coupon payment
CR = coupon rate
FV = face value of bond
CPY = number of coupon payments per year

Ex. Assume a bond with a $1000 face value pays a 10% coupon rate. What coupon does the issuer promise to pay bondholders if the coupons are paid semiannually (as most are)?

$$CPN = 50 = \frac{.1 \times 1000}{2}$$

B. Zero-Coupon Bonds

1. Yield to Maturity

Notes:

1) Yield to maturity = special name of IRR on bond
   => discount rate that sets present value of promised bond payments equal to current market price of bond
2) If a bond is risk-free, the yield to maturity is the same as IRR in chapter 4.
3) Do not really need the following equations. Can use equation (4.2) to solve for present value (to get price) or to solve for "r" to get YTM.

$$P = \frac{FV}{(1+YTM_n)^n} \tag{6.2}$$

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1 \tag{6.3}$$

where:

YTM_n = yield to maturity from holding the bond from today until matures on date n

Ex. Assume a zero-coupon bond pays $1000 when it matures 5 years from today and that the yield to maturity on the bond equals 4.5%. What is the price of the bond?

=> ***draw timeline***

$$P = \frac{\mathbf{1000}}{(\mathbf{1.045})^{\mathbf{5}}} = \mathbf{802.45}$$

Ex. Assume the price of the previous bond rises to $810. What is the yield to maturity on the bond?

=> ***draw timeline***

$$YTM_5 = \left(\frac{\mathbf{1000}}{\mathbf{810}}\right)^{\mathbf{1/5}} - \mathbf{1} = \mathbf{.04304}$$

2. Risk-free Interest Rates

=> the risk-free interest rate for a maturity of n years equals the yield to maturity on a zero-coupon risk-free bond that matures n years from today.

$$r_n = YTM_n \tag{6.4}$$

C. Coupon Bonds

=> Coupon bonds pay par at maturity. They also pay a coupon at maturity and pay a coupon every period (usually semiannually) before this.

Important: Assume semiannual coupons unless told otherwise.

Note: Do not really need the following equation. Can combine equations (4.2) and (4.9) or (4.12, if assume g = 0).

$$P = CPN \times \frac{1}{y}\left(1 - \left(\frac{1}{(1+y)^N}\right)\right) + \frac{FV}{(1+y)^N} \tag{6.5}$$

where:

y = return per coupon period on coupon bond
FV = face value of bond
N = number of coupons remaining (and coupon periods to maturity)

Notes:

1) In the text, footnote #3 discussing equation 6.5 is important. As finance majors, you need to know how to value bonds at all dates…not just at coupon dates.
2) YTM (an APR) = y x N
   You can calculate the effective annual rate from rate per coupon interval. But the rate normally quoted for bonds is the APR. To compare the returns on bonds with different coupon intervals, need to compare effective annual interest rates (APYs).

Ex. Assume a bond matures for $1000 six years from today and has a 7% coupon rate with semiannual coupons. What is the value of the bond today if the yield to maturity on the bond equals 8.5%?

=> ***draw timeline***

$$CPN = \mathbf{35} = \frac{.07 \times 1000}{2}$$

$$y = \mathbf{.0425} = \frac{.085}{2}$$

$$P = \mathbf{930.62} = \frac{35}{.0425}\left(1 - \left(\frac{1}{1.0425}\right)^{12}\right) + \frac{1000}{(1.0425)^{12}}$$

Ex. Assume a bond matures for $1000 seven years from today and had a 9.5% annual coupon rate (paid semiannually). What is the yield to maturity on the bond if the price today is $1050?

=> ***draw timeline***

$$CPN = \mathbf{47.5} = \frac{.095 \times 1000}{2}$$

$$P = \frac{47.5}{y}\left(1 - \left(\frac{1}{1+y}\right)^{14}\right) + \frac{1000}{(1+y)^{14}} = \mathbf{1050}$$

Using goal seek in Excel:

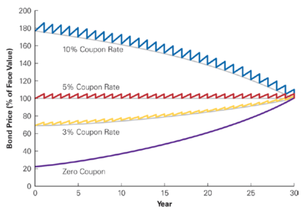
y = .04268
=> YTM = yield to maturity = **.0854 = .04268 x 2**

6.2 Dynamic Behavior of Bond Prices

A. Discounts and Premiums

Key issues:

1) coupon rate vs. yield to maturity
2) return on bond driven by coupons and change in price
3) over time, bond prices tend to move towards par value
4) bond prices rise between coupon payments and fall on coupon payments



=> Figure 6.1 from the textbook

5) prices rise if interest rate falls and prices fall if interest rate rises
   Reason: present value of future cash flows rise when interest rates fall and fall when interest rates rise

B. Time and Bond Prices

Key issues:

1) bond prices must eventually end up at par (+ coupon) just before maturity

   Note: See figure 6.1 above

2) if interest rates don't change, will earn yield to maturity over time hold bond

C. Interest Rate Changes and Bond Prices

Key issue: sensitivity of bond price to changes in interest rate depends on bond's duration

Note: Duration is basically the average maturity of the bond's cash flows (coupons and par value). So the longer the duration, the longer the average maturity of the bond's cash flow…and the more sensitive the bond to changes in interest rates.

Ex. Assume the interest rate for all maturities is 5% and that the face value of the bonds equals $1000.

Q1: What is the price today of a 5-year bond paying a 10% coupon rate with annual coupons?

=> ***draw timeline***

$$P = \frac{\mathbf{100}}{\mathbf{.05}}\left(\mathbf{1} - \left(\frac{\mathbf{1}}{\mathbf{1.05}}\right)^{\mathbf{5}}\right) + \frac{\mathbf{1000}}{(\mathbf{1.05})^{\mathbf{5}}} = \mathbf{1216.47}$$

Q2: What is the price today of a 30-year zero coupon bond?

=> ***draw timeline***

$$P = \frac{\mathbf{1000}}{(\mathbf{1.05})^{\mathbf{30}}} = \mathbf{231.38}$$

Q3: Which bond's price will have a larger percentage change if interest rates rise to 8%?

Note: Duration of 5-year bond is less than 5 years since some cash flows before maturity. Duration of 30-year bond is 30 years since only one cash flow.

Coupon bond price falls by 11% to $1079.85

$$P = \frac{\mathbf{100}}{\mathbf{.08}}\left(\mathbf{1} - \left(\frac{\mathbf{1}}{\mathbf{1.08}}\right)^{\mathbf{5}}\right) + \frac{\mathbf{1000}}{(\mathbf{1.08})^{\mathbf{5}}} = \mathbf{1079.85}$$

Zero-coupon bond price falls 57% to $99.38

$$P = \frac{\mathbf{1000}}{(\mathbf{1.08})^{\mathbf{30}}} = \mathbf{99.38}$$

Note: Price of a 5-year zero coupon bond falls 13% from 783.53 to 680.58

Note: duration of the zero-coupon bond is 5 years rather than less than 5 years for the coupon bond

1. Clean and Dirty Prices for Coupon Bonds

$$CP = DP - CPN\left(\frac{DSLC}{DICP}\right) \tag{6.A}$$

where:

CP = clean price
DP = dirty price = cash price
CPN = coupon
DSLC = days since last coupon
DICP = days in coupon period

Note: I will often use MSLC (months since last coupon) and MICP (months in coupon period) instead of DSLC and DICP

Note: Accrued interest is linear while the change in a bond's price over time is not (due to compounding). Thus the clean price will still have a very slight saw-tooth pattern over time if interest rates do not change.

Ex. Assume that a bond with an 8.5% coupon rate (semiannual coupons) matures ten years from today.

a. What is the value (and price) of the bond today (per $100 of face value) if the yield to maturity equals 5%?

Coupon = $\mathbf{4.25} = \frac{.085 \times 100}{2}$

Effective rate per coupon period $=\mathbf{.025} = \frac{.05}{2}$

=> ***draw timeline***

$$V_0 = \mathbf{127.2810} = \frac{4.25}{.025}\left(1 - \left(\frac{1}{1.025}\right)^{20}\right) + \frac{100}{(1.025)^{20}}$$

b. Assume that four months have elapsed (maturity is now 9 years and 8 months from today), but the yield to maturity on the bond has not changed. What is the value, dirty price, and clean price of the bond?

=> ***draw timeline***

$$V_{-4mo} = \frac{\mathbf{4.25}}{\mathbf{.025}}\left(\mathbf{1} - \left(\frac{\mathbf{1}}{\mathbf{1.025}}\right)^{\mathbf{20}}\right) + \frac{\mathbf{100}}{(\mathbf{1.025})^{\mathbf{20}}} = \mathbf{127.2810}$$

Note: No coupons have been paid and interest rates have not changed

=> the value one coupon-period before first coupon is unchanged.

$$V_0 = \mathbf{127.2810(1.025)^{4/6}} = \mathbf{129.3936}$$

Dirty price = 129.3936

Clean price = $\mathbf{129.3936 - 4.25\left(\frac{4}{6}\right)} = \mathbf{126.5603}$

Note: A more precise answer would use days rather than months. But in this case, we would really need to calculate the value of each coupon payment and the par value separately based on how many days from today each payment is made.

c. Assume that the bond matures 9 years and 8 months from today (as in "b", four months have elapsed), but that the yield to maturity has fallen to 4% instead of remaining at 5%. What is the value, dirty price, and clean price of the bond?

$$\text{dirty price} = V_0 = \left(\frac{\mathbf{4.25}}{\mathbf{.02}}\left(\mathbf{1} - \left(\frac{\mathbf{1}}{\mathbf{1.02}}\right)^{\mathbf{20}}\right) + \frac{\mathbf{100}}{(\mathbf{1.02})^{\mathbf{20}}}\right)(\mathbf{1.02})^{\mathbf{4/6}} = \mathbf{138.6086}$$

Clean price = $\mathbf{138.6086 - 4.25\left(\frac{4}{6}\right)} = \mathbf{135.7752}$

d. Assume that the bond matures 9 years and 8 months from today and that the clean price of the bond is $120. What is the yield to maturity on the bond?

=> draw timeline

Cash price = $\mathbf{120 + 4.25\left(\frac{4}{6}\right) = 122.8333}$

YTM: $\left(\frac{4.25}{y}\left(1-\left(\frac{1}{1+y}\right)^{20}\right)+\frac{100}{(1+y)^{20}}\right)(1+y)^{4/6} = 122.8333$

=> solving for y (using solver in Excel): y = .02885

=> yield to maturity (YTM) = .0577 = .02885 × 2

6.3 The Yield Curve and Bond Arbitrage

A. Replicating a Coupon Bond

Key issues:

1) coupon bonds can be replicated with zero-coupon bonds
2) can use Law of One Price to value coupon bonds

B. Valuing a Coupon Bond Using Zero-Coupon Yields

$$P = \frac{CPN}{1+YTM_1} + \frac{CPN}{(1+YTM_2)^2} + \cdots + \frac{CPN+FV}{(1+YTM_n)^n} \qquad (6.6)$$

C. Coupon Bond Yields

Key idea: yield to maturity on a coupon bond equals a weighted average of yields on the zero-coupon bonds that can be used to replicate the bond

Ex. Assume a risk-free bond with a 4% annual coupon rate (paid annually) matures five years from today for $1000. Assume also that the yield on risk-free, zero-coupon bonds varies by maturity as follows: 1-year = 2%, 2-year = 3%, 3-year = 4%, 4-year = 4.5%, 5-year = 5%.

a. What is the no-arbitrage price for the bond?
b. What is the yield to maturity on the bond if it trades at the no-arbitrage price?
c. What transactions would you need to undertake today to set up arbitrage if the price of the bond equals $970?

a. No-arbitrage price for bond

=> ***draw timeline***

Price = 960.89 = 39.216+37.704+35.560+33.542+814.867 = $\frac{40}{(1.02)^1}+\frac{40}{(1.03)^2}+\frac{40}{(1.04)^3}+\frac{40}{(1.045)^4}+\frac{1040}{(1.05)^5}$

b. $\left(\frac{40}{y}\left(1-\left(\frac{1}{1+y}\right)^5\right)+\frac{1000}{(1+y)^5}\right) = 960.89 \Rightarrow y = \mathbf{4.901}\%$ (goal seek in Excel)

Note: most of investment is in final cash flow which earns 5%
=> average return closest to 5%.

c. => arbitrage if price = 970: short bond and buy zero-coupon bonds

Table 1

	Cash Flows in year:					
Transaction (t = 0)	0	1	2	3	4	5
Short Bond	970	-40	-40	-40	-40	-1040
Buy 1-yr Zero	-39.216	40				
Buy 2-yr Zero	-37.704		40			
Buy 3-yr Zero	-35.560			40		
Buy 4-yr Zero	-33.542				40	
Buy 5-yr Zero	-814.867					1040
Total	9.111	0	0	0	0	0

6.4 Corporate Bonds

Key term: credit risk

A. Corporate Bond Yields

Ex. Assume a corporate bond matures 5 years from today and pays an annual coupon rate of 4% (annual coupons).

Key issues:

=> the yield to maturity is based on promised rather than expected cash flows
=> the expected return is less than the yield to maturity if there is a chance of default

1. No Default

=> same yield as Treasuries

Ex. Assume the bonds are risk free and that the risk-free interest rate equals 6%. What is the price and yield to maturity on the bonds?

Price: $\frac{40}{.06}\left(1-\left(\frac{1}{1.06}\right)^5\right)+\frac{1000}{(1.06)^5}=915.75$

YTM: $\frac{40}{y}\left(1-\left(\frac{1}{1+y}\right)^5\right)+\frac{1000}{(1+y)^5}=915.75$

=> y = .06

2. Certain Default and Payoff

=> bond can still be risk-free if know what will pay in default
=> expected return (based on expected cash flows) < yield to maturity (based on promised cash flows)

Ex. Assume a corporate bond matures 5 years from today and pays an annual coupon rate of 4% (annual coupons). Assume also that the bond will pay the first four coupons in full but that we know for sure that the firm will only pay 75% of the final coupon and par value at maturity as the firm defaults. What is the price and yield to maturity on the bond?

Price: $\frac{\mathbf{40}}{\mathbf{.06}}\left(\mathbf{1}-\left(\frac{\mathbf{1}}{\mathbf{1.06}}\right)^{\mathbf{4}}\right)+\frac{\mathbf{.75}\times(\mathbf{1040})}{(\mathbf{1.06})^{\mathbf{5}}}=\mathbf{721.4656}$

Note: Bondholders still require only a 6% return since risk-free

YTM: $\frac{40}{y}\left(1-\left(\frac{1}{1+y}\right)^{5}\right)+\frac{1000}{(1+y)^{5}}=721.4656$

=> y = .1166

=> The yield to maturity equals 11.66%, but the return only equals 6% because at maturity firm will default for sure and only pay 75% of what is due.

3. Risk of Default

=> value = present value of expected cash flows discounted at an appropriate cost of capital for securities with equivalent risk

Ex. Assume a corporate bond matures 5 years from today and pays an annual coupon rate of 4% (annual coupons). Assume also that the firm will pay all coupons before the bond matures but that there is an 80% chance that the bond will pay in full the final coupon and par value at maturity, but a 20% chance that the firm will default on the bond at maturity. If the firm defaults, it will only pay 60% of the final coupon and par value. Given the risk, investors demand a 1% risk premium (over 6%) on the bond. What is the value and yield to maturity on the bond?

Payoff in default = 624 = $\mathbf{.6 \times 1040}$

$E(CF) = \sum p \times CF$

where:

$E(CF)$ = expected cash flow
p = probability of cash flow
CF = possible cash flow

Expected cash flow = 956.8 $\mathbf{= .8 \times 1040 + .2 \times 624}$

Price: $\mathbf{\frac{40}{.07}\left(1 - \left(\frac{1}{1.07}\right)^4\right) + \frac{956.8}{(1.07)^5} = 817.6736}$

YTM: $\frac{40}{y}\left(1 - \left(\frac{1}{1+y}\right)^5\right) + \frac{1000}{(1+y)^5} = 817.6736$

=> y = .08644

=> Yield to maturity will equal 8.644%, but expected return only equals 7% because firm might default and not pay full amount due at maturity.

B. Corporate Bond Ratings

Rating companies: Standard & Poor's, Moody's and Fitch (not mentioned in text).
Bond categories: investment-grade, speculative (or junk or high-yield)

Rating*	Description (Moody's)
Investment Grade Debt	
Aaa/AAA	Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.
Aa/AA	Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.
A/A	Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.
Baa/BBB	Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.
Speculative Bonds	
Ba/BB	Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.
B/B	Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.
Caa/CCC	Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.
Ca/CC	Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.
C/C, D	Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

=> Table 6.4 from textbook

Note: Markets do not typically react much to changes in credit ratings

Reason: Bond prices tend to react to changes in the probability of default before bond ratings do.

6.5 Sovereign Bonds

Note: One other difference between sovereign debt and corporate debt is the lack of collateral with sovereign debt

=> when a company defaults, assets can (and usually are) seized by creditors.
=> this is less likely to be the case with a country

Chapter 6 Appendix

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