Chapter 5: Interest Rates

5.1. Interest Rate Quotes

A. Key ideas

1. Compounding: **earn interest on interest because interest is added to the balance.**

2. Interest rates typically quoted in one of two basic ways:

   a. Annual Percentage Rates [APR] – **annual interest rate that ignores the impact of compounding**

   Note: The Truth in Lending Act of 1968 requires lenders to report this rate

   b. Effective interest rate [r(t)] – **interest rate that includes the impact of compounding**
   t = **time frame of the interest rate in years**
   => **actual interest rate per period t**

   Ex.
   $r\left(\frac{1}{12}\right)$ = effective monthly rate
   $r(1)$ = effective annual interest rate

   Note: r(1) is also called 1) the APY (Annual Percentage Yield) because of the Truth in Savings Act of 1991 and 2) the EAR (effective annual rate).

   Ex. Assume given two interest rates for an account. The APR is 6% and the APY is 6.17%.
   => if deposit $100 for a year, end up with $106.17 not $106.

3. **With the exception of continuous compounding, use only effective interest rates in time value of money calculations**

4. **When calculating the PV or FV of a single cash flow, can use any effective rate**

5. **When calculating the PV or FV of a series of cash flows, must use the effective rate that matches the time between the cash flows.**

   Ex. monthly cash flows => **must use effective monthly rate**

B. Converting interest rates

1. Converting APRs to effective rates

$$r(t) = \frac{APR}{k} \tag{5.2}$$

where:

k = **number of compounding periods per year**
t = time frame of the interest rate in years = 1/k
Note: **any time given an APR, must start with this equation**

2. Converting between effective interest rates for different time periods

$$r(t) = (1 + r)^n - 1 \tag{5.1}$$

Usefulness: convert to an effective rate that matches the time between cash flows

Notes:

1) n = conversion ratio
2) to convert to a longer period, **n > 1**
3) to convert to a shorter period, **n < 1**

Ex. If want an interest rate for a period that is twice as long as the one you start with, n = **2**

Ex. If want an interest rate for a period that is twelve times as long as the one you start with, n = **12**

Ex. If want an interest rate for a period that is one-fourth as long as the one you start with, n = **1/4**

Ex. Assume an APR of 6% per year with semiannual compounding. What effective annual interest rate and effective monthly interest rate is equivalent to an APR of 6% per year with semiannual compounding?

$r\left(\frac{1}{2}\right) = \frac{\mathbf{.06}}{\mathbf{2}} = \mathbf{.03}$

=> ***effective semiannual rate (half a year) is 3%***

$r(1) = (\mathbf{1.03})^{\mathbf{2}} - \mathbf{1} = \mathbf{.0609}$

$r\left(\frac{1}{12}\right) = (\mathbf{1.03})^{\mathbf{1/6}} - \mathbf{1} = \mathbf{.004939}$

Note: $r\left(\frac{1}{2}\right) = .03$, $r(1) = .0609$, and $r\left(\frac{1}{12}\right) = .004939$ are equivalent

=> ***end up with same amount of money at the end***

Ex. If invest \$100 for a year, then your account balance at the end of the year equals:

$$V_1 = \mathbf{100(1.03)^2 = 100(1.0609) = 100(1.004939)^{12} = 106.09}$$

Ex. Eight months from today you want to make the first of 12 quarterly withdrawals from a bank account. Your first withdrawal will equal \$10,000 and each subsequent withdrawal will grow by 1% each. How much do you need to deposit today if the account pays an APR of 9% with monthly compounding?

Steps: 1) timeline; 2) pattern (annuity); 3) equation; Q: PV or FV? Q: Where end up on timeline?

=> ***draw timeline***

$r\left(\frac{1}{12}\right) = \frac{\mathbf{.09}}{\mathbf{12}} = \mathbf{.0075}$

$r\left(\frac{1}{4}\right) = (\mathbf{1.0075})^{\mathbf{3}} - \mathbf{1} = \mathbf{.022669}$

$$V_{5mo} = \frac{\mathbf{10{,}000}}{\mathbf{.022669 - .01}}\left(\mathbf{1} - \left(\frac{\mathbf{1.01}}{\mathbf{1.022669}}\right)^{\mathbf{12}}\right) = \mathbf{109{,}666.07}$$

Steps: 2) pattern (single); 3) equation; Q: PV or FV? Q: Where end up on timeline?

$$V_0 = \frac{\mathbf{109{,}666.07}}{\mathbf{(1.0075)^5}} = \mathbf{105{,}644.52}$$

Ex. What if you want to make the first withdrawal one month from today (and nothing else changes)?

Q: Will the amount you deposit be larger or smaller if the 1st withdrawal is one month from today instead of eight months? Why? **Larger**

Steps: 1) timeline; 2) pattern (annuity); 3) equation; Q: PV or FV? Q: Where end up on timeline?

=> ***draw timeline***

$r\left(\frac{1}{12}\right) = .0075; r\left(\frac{1}{4}\right) = .022669; V_{-2mo} = 109{,}666.07$

Q: Why?

$V_0 = \mathbf{109,666.07(1.0075)^2 = 111,317.23}$

Ex. A bond matures for $1000 three years and ten months from today. The annual coupon on the bond equals $60 but coupons are paid semiannually. What is the value of the bond if it earns a return of 8% per year?

Steps: 1) timeline; 2) pattern (annuity and single); 3) equation; Q: PV or FV? Q: Where end up on timeline?

=> ***draw timeline***

$r\left(\frac{1}{2}\right) = \mathbf{(1.08)^{1/2} - 1 = .03923}$

Coupons:

$V_{-2mo} = \frac{\mathbf{30}}{\mathbf{.03923}}\left(\mathbf{1} - \left(\frac{\mathbf{1}}{\mathbf{1.03923}}\right)^{\mathbf{8}}\right) = \mathbf{202.6257}$

$V_0 = \mathbf{202.6257(1.08)^{2/12} = 205.2415}$

Par

3) equation; Q: <u>PV</u> or FV?

$V_0 = \frac{\mathbf{1000}}{\mathbf{(1.08)^{3\frac{10}{12}}}} = \mathbf{744.5187}$

Price = **205.25 + 744.52 = 949.76**

Calculator:

$V_{-2\ mo}$: 30 = PMT, 1000 = FV, 8 = N, 3.923 = I% => PV = 937.6555

V_0: 937.6555 = PV, 8 = I%, 2/12 = N => FV = 949.76

5.2 Application: Discount Rates and Loans

A. Computing Loan Payments

An important statement you might overlook: "When the compounding interval for the APR is not stated explicitly, it is equal to the interval between payments."

B. Computing the Outstanding Loan Balance

=> calculate present value of remaining payments

5.3 Determinants of interest rates

A. Inflation

Nominal interest rate: **growth rate of money**

Real interest rate: **growth rate of purchasing power**

Ex. Assume the nominal interest rate is 6% per year and that the real interest rate is 4% per year

=> after one year you will:

1) **have 6% more dollars**

2) **be able to buy 4% more stuff**

1. Basic idea: **investors care about real rather than nominal interest rates**
2. **if investors expect inflation to increase, nominal rates will increase**

   => **compensates investors for their loss of purchasing power**

3. Converting between nominal and real interest rates

$$r_r = \frac{r-i}{1+i} \tag{5.5}$$

where:

r_r = real interest rate

r = nominal interest rate

i = inflation rate

Note: can use expected or realized rates

Ex. Assume that the nominal interest rate is 6% per year and that inflation is 5% per year. What is the real interest rate?

r_r = **.00952 =** $\mathbf{\frac{.06-.05}{1.05}}$

Assume also that you can buy a ton of cocoa for $2200. If you invest the $2200 at 6%, you end up with $2332 in a year, but the cost of a ton of cocoa has risen to $2310. So you can buy 1.0095 tons in a year.

Calculations:

2332 = **2200 (1.06)**

2310 = **2200(1.05)**

1.0095 = **2332/2310 = 1+** $\mathbf{r_r}$

Note: the difference between the nominal rate and the inflation rate is a pretty good approximation of the real rate if inflation is low (1% = 6% - 5% in example)

B. The Fed

Basic idea: The Federal Reserve lowers or raises interest rates to stimulate or cool off the economy.

Key: **if lower (raise) interest rates, more (fewer) investments worthwhile since NPVs rise (fall)**

C. Maturity

Basic ideas:

1) interest rates vary by maturity

Ex. You can see how interest rates on U.S. Treasuries vary with maturity by googling Treasury rates or by following this link: https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

Note: credit default swap prices now indicate a nonzero chance of default by the U.S. Treasury

2) long-term rates usually exceed short-term rates

3) long-term rates reflect what investors expect will happen to short-term rates in the future

5.4 Risk and Taxes

A. Taxes

Basic idea: **investors care about after-tax returns**

=> **the higher the tax rates, the higher the return investors will demand**

After-tax interest rate: $r_{AT} = r - (\tau \times r) = r(1 - \tau)$ (5.8)

Where:

r_{AT} = after-tax interest rate

r = before-tax interest rate

τ = tax rate

B. Risk

Basic idea: **the greater the risk, the higher the interest rate**

5.5 The Opportunity Cost of Capital

Opportunity cost of capital (or simply cost of capital): best available expected return offered in the market on an investment of comparable risk and term to the cash flow being discounted.

Chapter 5 Appendix

A. Discount Rates for a Continuously Compounded APR

Key issue: converting between APR and effective annual interest rate when interest is compounded continuously

Note: Can't use equation 5.2 since k = ∞

$$r(1) = e^{APR} - 1 \tag{5A.1}$$

$$\text{APR} = \ln(1+r(1)) \tag{5A.2}$$

Ex. Assume a bank pays an APR of 5% with continuous compounding. What is the effective annual interest rate?

$$r(1) = \boldsymbol{e^{.05} - 1 = .05127}$$

Excel: = **exp(.05) – 1**

B. Continuously Arriving Cash Flows

=> skip this section