Chapter 17 Supplement: Steps and explanations in some of Chapter 17's equations

A. Deriving Equation 17.1 considering a long-term investor

1. Assume an investor is planning to sell a stock that is about to go ex-dividend. Assume also that the investor bought at price P_{buy}.

   a. If the investor sells the stock just before it goes ex-dividend, the payoff =
   $P_{cum} - \tau_g (P_{cum} - P_{buy})$

   where:

   P_{cum} = price of stock just before it goes ex-dividend
   τ_g = capital gains tax rate

   Note: lose τ_g of capital gain to taxes

   b. If the investor sells the stock just after it goes ex-dividend, the payoff =
   $Div(1 - \tau_d) + P_{ex} - \tau_g (P_{ex} - P_{buy})$

   where:

   P_{ex} = price of stock just after it goes ex-dividend
   τ_d = dividend tax rate

   Notes:

   1) lose τ_g of capital gain to taxes
   2) lose τ_d of dividend to taxes
   3) $P_{ex} < P_{cum}$

2. For the investor to be indifferent, the payoffs must be equal

   Note:

   => if payoff from selling before goes ex-div higher, all investors will want to sell before goes ex-div driving P_{cum} lower and P_{ex} higher.
   => if payoff from selling ex-div higher, all investors will wait to sell driving P_{cum} higher and P_{ex} lower.

   => $P_{cum} - \tau_g (P_{cum} - P_{buy}) = Div(1 - \tau_d) + P_{ex} - \tau_g (P_{ex} - P_{buy})$
   $P_{cum} - \tau_g P_{cum} + \tau_g P_{buy} = Div(1 - \tau_d) + P_{ex} - \tau_g P_{ex} + \tau_g P_{buy}$

   $P_{cum}(1 - \tau_g) = Div(1 - \tau_d) + P_{ex}(1 - \tau_g)$ Note: subtract $\tau_g P_{buy}$ from both sides

$$P_{cum}(1-\tau_g) - P_{ex}(1-\tau_g) = Div(1-\tau_d)$$
$$(P_{cum} - P_{ex})(1-\tau_g) = Div(1-\tau_d)$$

B. Deriving Equation 17.2 from 17.1

$$(P_{cum} - P_{ex})(1-\tau_g) = Div(1-\tau_d) \qquad (17.1)$$

$$P_{cum} - P_{ex} = Div\left(\frac{1-\tau_d}{1-\tau_g}\right)$$

$$P_{cum} - P_{ex} = Div\left(1 - 1 + \frac{1-\tau_d}{1-\tau_g}\right)$$

$$P_{cum} - P_{ex} = Div\left(1 - \frac{1-\tau_g}{1-\tau_g} - \frac{\tau_d - 1}{1-\tau_g}\right)$$

$$P_{cum} - P_{ex} = Div\left(1 - \frac{1-\tau_g + \tau_d - 1}{1-\tau_g}\right)$$

$$P_{cum} - P_{ex} = Div\left(1 - \frac{\tau_d - \tau_g}{1-\tau_g}\right)$$

$$P_{cum} - P_{ex} = Div(1-\tau_d^*)$$

where: $\tau_d^* = \left(\frac{\tau_d - \tau_g}{1-\tau_g}\right)$

C. Steps between (17.5) and (17.6)

$$P_{retain} = 100 \times \frac{(1-\tau_c)(1-\tau_d)}{(1-\tau_i)} \qquad (17.5)$$

$$= 100 \times \frac{(1-\tau_c)(1-\tau_d)(1-\tau_g)}{(1-\tau_i)(1-\tau_g)}$$

Note: can multiply top and bottom by same #

$$= 100 \times \frac{(1-\tau_d)}{(1-\tau_g)} \times \frac{(1-\tau_c)(1-\tau_g)}{(1-\tau_i)}$$

$$= P_{cum} \times \frac{(1-\tau_c)(1-\tau_g)}{(1-\tau_i)}$$

Note: $P_{cum} = 100 \times \left(\frac{1-\tau_d}{1-\tau_g}\right)$ from (17.4)

$$= P_{cum} \times \left(1 - 1 + \frac{(1-\tau_c)(1-\tau_g)}{(1-\tau_i)}\right)$$

$$= P_{cum} \times \left(1 - \left(1 - \frac{(1-\tau_c)(1-\tau_g)}{(1-\tau_i)} \right) \right)$$

$$= P_{cum} \times \left(1 - \tau^*_{retain}\right) \qquad (17.6)$$

$$\text{where: } \tau^*_{retain} = 1 - \frac{(1-\tau_c)(1-\tau_g)}{(1-\tau_i)} \qquad (17.7)$$