Chapter 15: Supplement

Proof of Equation (15.7) and (15.8)

Let:

CF_A = cash flow generated by firm's assets each year
τ_c = tax rate on corporate income
τ_i = tax rate on interest income for individuals
τ_e = tax rate on equity income for individuals
D = permanent debt level
r_U = after-tax return demanded by stockholders of unlevered firm
r_D = interest rate paid on debt

Note: the after-tax return for bondholders = $r_D(1-\tau_i)$
=> bondholders will discount the after-tax payments they receive at $r_D(1-\tau_i)$

If firm has no debt:

CF to stockholders = $CF_A\,(1-\tau_c)(1-\tau_e)$ (1)

$$V^U = \frac{CF_A(1-\tau_c)(1-\tau_e)}{r_u} \quad (2)$$

If firm has debt:

CF to stockholders = $(CF_A - r_D D)(1-\tau_c)(1-\tau_e)$ (3)

CF to debt holders = $r_D D(1-\tau_i)$ (4)

=> CF to investors = $(CF_A - r_D D)(1-\tau_c)(1-\tau_e) + r_D D(1-\tau_i)$ (5)

$= CF_A(1-\tau_c)(1-\tau_e) - r_D D(1-\tau_c)(1-\tau_e) + r_D D(1-\tau_i)$ (6)
$= CF_A(1-\tau_c)(1-\tau_e) + r_D D((1-\tau_i) - (1-\tau_c)(1-\tau_e))$ (7)

$$V^L = \frac{CF_A(1-\tau_c)(1-\tau_e)}{r_u} + \frac{r_D D\left((1-\tau_i) - (1-\tau_c)(1-\tau_e)\right)}{r_D(1-\tau_i)} \quad (8)$$

Notes:

1) equivalent to take the present value of an entire cash flow or to take the present value of the parts and add up the present values.
2) since first piece of cash flow in (5) is same as the cash flow to the stockholders of an unlevered firm (1), should use same rate as for an unlevered firm (r_U)

3) since the second piece of the cash flow is a fixed percent of the interest payments ($r_D D$), you can use the same rate on this second piece as you can on the interest payments themselves…and the appropriate after-tax discount rate is thus $r_D(1-\tau_i)$

Substituting equation (2) for the first part of (8) and canceling out r_D we get:

$$V^L = V^U + \frac{D\left(\left(1-\tau_i\right)-\left(1-\tau_c\right)\left(1-\tau_e\right)\right)}{\left(1-\tau_i\right)} = V^U + \left(\frac{\left(1-\tau_i\right)-\left(1-\tau_c\right)\left(1-\tau_e\right)}{\left(1-\tau_i\right)}\right)D \tag{9}$$

$$\Rightarrow V^L = V^U + \left(1-\frac{\left(1-\tau_c\right)\left(1-\tau_e\right)}{\left(1-\tau_i\right)}\right)D \tag{10}$$

$$\Rightarrow V^L = V^U + \tau^* D \tag{15.8}$$

$$\text{where: } \tau^* = 1-\frac{\left(1-\tau_c\right)\left(1-\tau_e\right)}{\left(1-\tau_i\right)} \tag{15.7}$$