Chapter 15: Debt and Taxes

I. Basic Ideas

1. Corporate Taxes

=> **interest expense is tax deductible**
=> **as debt increases, corporate taxes fall**
=> **incentive to fund the firm with debt**

2. Personal taxes

=> equity income is usually taxed at a lower rate than interest income
=> **incentive to fund the firm with equity**

II. Corporate Income Taxes

A. Interest Tax Shield

Ex. Assume you are starting a firm that will generate a pre-tax cash flow of $100,000 per year. What are your after-tax cash flows if you call your investment equity and if you call your investment a loan? Assume the corporate tax rate is 35% and that your firm pays out all of its cash flows either as dividends or interest.

If paid as a dividend: $65,000 = **100,000 – .35(100,000)**
If paid as interest: $100,000

=> *extra $35,000 per year if call investment a loan*

Q: Where does the $35,000/year come from?
Q: If we only consider corporate taxes, how should firms be funded?

Let:

CF^L = cash flows to investors with leverage
CF^U = cash flows to investors without leverage (unlevered)
τ_C = corporate tax rate

=> $CF^L = CF^U$ + Interest Tax Shield

=> interest tax shield = reduction in taxes because issue debt = $\tau_c \times$ *interest expense*

Ex. CF^L = 100,000 = **65,000 + .35(100,000)**

B. Interest Tax Shield and Firm Value

Let:

V^L = value of levered firm (firm with debt)
V^U = value of unlevered firm (firm with no debt)

$V^L = V^U$ + PV(Interest Tax Shield) (15.2)

C. The Interest Tax Shield with Permanent Debt

1. Assumptions:

1) Firm plans to keep a constant dollar amount of debt forever
2) Firm's marginal tax rate is fixed

2. PV of Tax Shield of Permanent Debt

PV(Interest Tax Shield) = PV(τ_C x Future Interest Payments)

$$= \tau_c \times \text{PV(Future Interest Payments)}$$
$$= \tau_c \times \frac{Interest\ Payment}{r}$$
$$= \tau_c \times \frac{r \times D}{r}$$
$$= \tau_c \times D \quad (15.4)$$

where:

r = interest rate on debt
D = market value of firm's outstanding debt

$\Rightarrow V^L = V^U + \tau_C D$ (15.A)

Ex. Assume firm is currently funded with 100% equity and has a market value of $500,000. What will the firm be worth if the corporate tax rate is 35%, and the firm issues $200,000 of permanent debt and uses the proceeds to repurchase stock?

V^L = 570,000 = **500,000 + .35(200,000)**
=> U.S. corporate tax code gives firms an incentive to issue debt
Q: Where does the extra $70,000 come from?

D. The Weighted Average Cost of Capital with Taxes

From Chapter 14:
E = market value of firm's outstanding equity
D = market value of firm's outstanding debt
r_E = cost of capital for levered equity
r_D = cost of capital for firm's debt

Basic idea: **tax savings reduce the cost of borrowing**

=> $r_{AT} = r_D(1- \tau_C)$ (15.B)
where: r_{AT} = after-tax cost of borrowing

Note: Equation 15.B can also be used to calculate the after-tax returns for investors if use the investor's tax rate rather than τ_C

$$r_{WACC} = \left(\frac{E}{E+D}\right) r_E + \left(\frac{D}{E+D}\right) r_D(1 - \tau_C) \quad (15.5)$$

Ex. Assume that the market value of a firm's equity is $300,000 and that the market value of its debt is $200,000. Assume also that the cost of equity is 12%, that the cost of debt is 5%, and that the corporate tax rate is 35%. What is the cost of capital for the firm if interest is not tax deductible and if it is?

$r_{WACC}(\text{not tax deductible}) = .092 = \left(\frac{300{,}000}{500{,}000}\right).\mathbf{12} + \left(\frac{200{,}000}{500{,}000}\right).\mathbf{05}$
$r_{WACC}(\text{tax deductible}) = .085 = \left(\frac{300{,}000}{500{,}000}\right).\mathbf{12} + \left(\frac{200{,}000}{500{,}000}\right).\mathbf{05}(\mathbf{1}-.\mathbf{35}) =$
$.\mathbf{6}(.\mathbf{12})+.\mathbf{4}(.\mathbf{0325})$

Q: What happens to the value of the firm today if the cost of capital is lower?
Increases

III. Personal Income Taxes

A. Personal Taxes and the Interest Tax Shield

=> main issue for capital structure: personal taxes are usually lower for equity income than debt income

=> **as corporate debt rises, personal taxes increase**

B. Valuing the Interest Tax Shield with Personal Taxes

$$V^L = V^U + \tau^* D \quad (15.8)$$

where:

$$\tau^* = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)} \quad (15.7)$$

τ^* = effective tax advantage of debt
τ_c = tax rate on corporate income
τ_e = tax rate on personal equity income,
τ_i = tax rate on personal interest income

Note: see Chapter 15 supplement for proof

Ex. Assume that $\tau_C = .35$, $\tau_i = .3$, and $\tau_e = .3$

$$\tau^* = .35 = \mathbf{1} - \frac{\mathbf{(1-.35)(1-.3)}}{\mathbf{(1-.3)}}$$

=> $V^L = \mathbf{V^U + .35D}$

Note: if $\tau_e = \tau_i$: **personal taxes cancel out**

Ex. Assume that $\tau_C = .35$, $\tau_i = .3$, and $\tau_e = .15$

$$\tau^* = .2107 = \mathbf{1} - \frac{\mathbf{(1-.35)(1-.15)}}{\mathbf{(1-.3)}}$$

=> $V^L = \mathbf{V^U + .2107D}$

Note: if $\tau_i > \tau_e$: **tax loss at personal level reduces tax benefit at corporate level**

Note: Difficult to calculate τ^* for several reasons

1. investors don't have to realize capital gains and can offset gains with losses
2. mix of dividends and capital gains varies by firm
3. returns on retirement accounts untaxed
4. tax rates vary across investors

IV. Optimal Capital Structure with Taxes

A. Limits to the Tax Benefit of Debt

1. Key => **debt has a tax benefit to the firm only if have enough earnings to deduct the interest**

   => **if can't deduct the interest and increase debt, taxes rise due to higher personal taxes**

2. Excess leverage: firm does not have enough earnings to deduct the interest

3. Tax impact of excess leverage

   => corporate taxes are not reduced

   => $\tau_c = 0$

   => $\tau_{ex}^* = 1 - \frac{(1-0)(1-\tau_e)}{(1-\tau_i)} = \frac{\tau_e - \tau_i}{(1-\tau_i)}$

   where τ_{ex}^* = net tax disadvantage of excessive debt

   => **tax loss if issue debt and $\tau_e < \tau_i$**

B. Optimal Leverage with Risk-free Earnings Before Interest and Taxes (EBIT)

Let $\tau^* = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)}$ (15.7)

Where: $\tau^*$is the effective tax advantage of debt

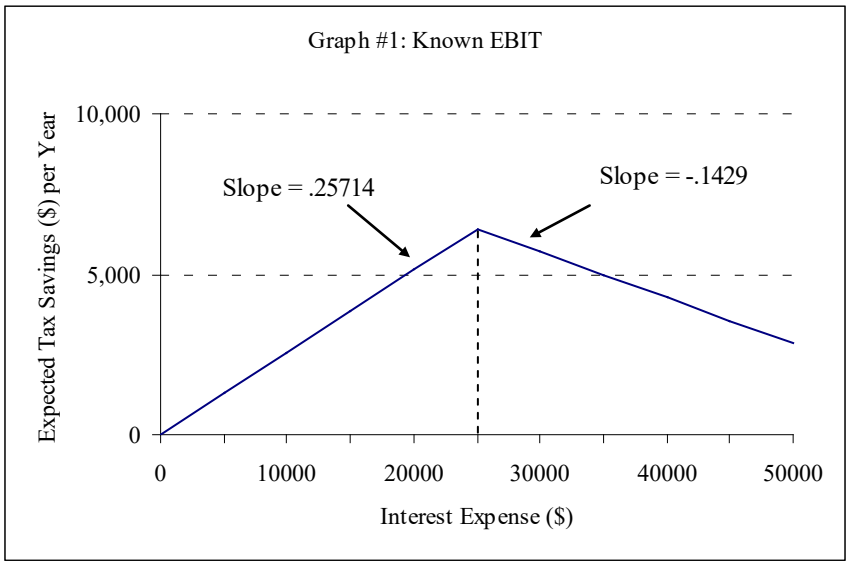
General idea: **issue debt as long as τ^* positive**

Example: Assume that a firm's riskless EBIT will be $25,000 per year forever. Assume also that the corporate tax rate is 35%. Finally, assume that the tax rate for individual interest income is 30% and that the tax rate on individual equity income is 20%.

=> if interest < $25,000 per year, $\tau^* = .25714 = \mathbf{1} - \frac{(\mathbf{1}-.\mathbf{35})(\mathbf{1}-.\mathbf{2})}{(\mathbf{1}-.\mathbf{3})}$

=> if interest $\geq$ $25,000 per year, $\tau^*_{ex} = -.1429 = \mathbf{1} - \frac{(\mathbf{1}-\mathbf{0})(\mathbf{1}-.\mathbf{2})}{(\mathbf{1}-.\mathbf{3})} = \frac{.\mathbf{2}-.\mathbf{3}}{(\mathbf{1}-.\mathbf{3})}$

Q: What do these numbers tell us?



Optimal debt: **set interest equal to EBIT**

=> no corporate taxes paid

=> optimal annual interest in example = **$25,000**

C. Optimal Leverage with Risky Earnings (EBIT)

=> **as interest expense increases, chance of deducting the interest falls**

=> use $E(\tau_C)$ rather than τ_C in equation 15.7

$E(\tau_C)$ = expected corporate tax savings from interest

= **probability of using tax shield x τ_C**

=> $\tau^* = 1 - \frac{(1-E(\tau_c))(1-\tau_e)}{(1-\tau_i)}$ (15.7B)

Key: issue debt as long as τ^* positive

Example: Assume that the corporate tax rate is 35%, that the personal tax rate is 30% on interest income and 20% on equity income. Assume also the following probability distribution for the firm's EBIT.

(15.7B) $\tau^* = 1 - \frac{(1-E(\tau_c))(1-\tau_e)}{(1-\tau_i)}$

EBIT	Prob.
10,000	.05
15,000	.10
20,000	.10
25,000	.15
30,000	.20
35,000	.20
40,000	.10
45,000	.10

Q: What is the probability of deducting interest of:

up to \$10,000? 100% => can deduct even if have lowest possible EBIT (10,000)

up to \$15,000? 95% => 95% chance of having EBIT of \$15,000 or higher =
.1+.1+.15+.2+.2+.1+.1 = 1 - .05

up to \$20,000? 85% => 85% chance of having EBIT of \$20,000 or higher =
.1+.15+.2+.2+.1+.1 = 1 - .05 - .1

up to \$25,000? 75% = .15 + .2 + .2 + .1 + .1 = 1 - .05 - .1 - .1

up to \$30,000? 60%

up to \$35,000? 40%

up to \$40,000? 20%

up to \$45,000? 10%

more than \$45,000? No chance since highest possible EBIT is \$45,000

Q: What is E(τ_C) and τ^* for interest paid equal to:

$0 to $10,000?

=> E(τ_C) = .35 = **1 x .35**
=> $\tau^* = .25714 = \mathbf{1} - \frac{(\mathbf{1}-.\mathbf{35})(\mathbf{1}-.\mathbf{2})}{(\mathbf{1}-.\mathbf{3})}$

Q: Issue debt if interest currently less than $10,000? yes

$10,001 to $15,000?

=> E(τ_C) = .3325 = **.95(.35)**
=> $\tau^* = .23714 = \mathbf{1} - \frac{(\mathbf{1}-.\mathbf{3325})(\mathbf{1}-.\mathbf{2})}{(\mathbf{1}-.\mathbf{3})}$

Q: Issue debt if interest currently between $10,001 and $15,000? yes

$15,001 to $20,000?

=> E(τ_C) = .2975 = **.85(.35)**
=> $\tau^* = .19714 = \mathbf{1} - \frac{(\mathbf{1}-.\mathbf{2975})(\mathbf{1}-.\mathbf{2})}{(\mathbf{1}-.\mathbf{3})}$

Q: Issue debt if interest currently between $15,001 and $20,000? yes

$20,001 to $25,000?

=> E(τ_C) = .2625 = .75(.35)
=> $\tau^* = .14714 = 1 - \frac{(1-.2625)(1-.2)}{(1-.3)}$
Q: Issue debt if interest currently between $20,001 and $25,000? yes

$25,001 to $30,000?

=> E(τ_C) = .21 = .6(.35)
=> $\tau^* = .09714 = 1 - \frac{(1-.21)(1-.2)}{(1-.3)}$
Q: Issue debt if interest currently between $25,001 and $30,000? yes

$30,001 to $35,000?

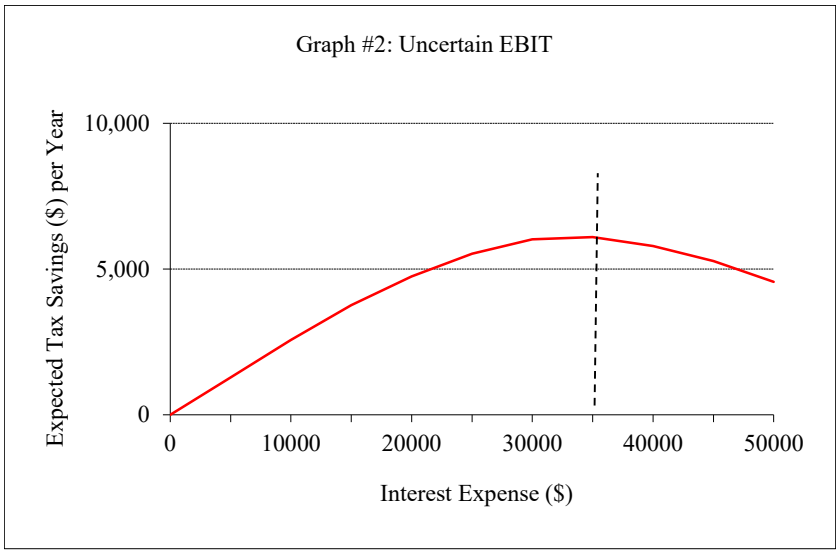
=> E(τ_C) = .14; $\tau^* = .01714$
Q: Issue debt if interest between $30,001 and $35,000? yes

$35,001 to $40,000: $E(\tau_C) = .07$; $\tau^* = -.06286$
Q: Issue debt if interest between $35,001 and $40,000? no

$40,001 to $45,000? $E(\tau_C) = .035$; $\tau^* = -.10286$
Q: Issue debt if interest between $40,001 and $45,000? no

$45,001 or greater? $E(\tau_C) = 0$; $\tau^* = -.14286 = \tau^*_{ex}$
Q: Issue debt if interest exceeds $45,000? no

=> optimal annual interest = $35,000



D. Growth and Debt

Note: Assume two firms have the same distribution of possible earnings (EBIT). Assume also that one firm is growing rapidly while the other is not growing.

1) **The tax-optimal debt for the firms will be <u>the same</u>**
2) **The value of the high-growth firm's equity will be <u>higher</u>**

Result: **The debt ratio of the high growth firm will be lower**