Chapter 14: Supplement

Leverage, Risk, and Return

Let:

E = market value of the firm's outstanding equity

D = market value of the firm's outstanding debt

 β_E = beta of firm's levered equity

 β_D = beta of firm's debt

 β_U = beta of firm's unlevered equity (if it has no debt)

- 1. Leverage, risk, and the cost of equity capital
 - 1) Unlevered equity is equivalent to a portfolio of debt and levered equity
 - 2) The beta of a portfolio is a weighted average of the portfolio's betas

$$\Rightarrow \beta_U = \left(\frac{E}{E+D}\right) \beta_E + \left(\frac{D}{E+D}\right) \beta_D \tag{14.9}$$

$$\Rightarrow$$
 solving for E: $\beta_E = \beta_U + \frac{D}{E} (\beta_U - \beta_D)$ (14.10)

=> since $\beta_D < \beta_U$, β_E increases as increase leverage (increase debt and reduce equity)

3) Impact of leverage on equity cost of capital:

$$r_{E} = r_{f} + \beta_{E} \left(E(R_{Mkt}) - r_{f} \right)$$

$$r_{E} = r_{f} + \left(\beta_{U} + \frac{D}{E} (\beta_{U} - \beta_{D}) \right) \left(E(R_{Mkt}) - r_{f} \right)$$
(12.4)

Note: substitute 14.10

$$r_E = r_f + \beta_U \left(E(R_{Mkt}) - r_f \right) + \frac{D}{E} \left(\beta_U \left(E(R_{Mkt}) - r_f \right) - \beta_D \left(E(R_{Mkt}) - r_f \right) \right)$$

Note: distributive property

$$r_{E} = r_{f} + \beta_{U} \Big(E(R_{Mkt}) - r_{f} \Big) + \frac{D}{F} \Big(\Big(r_{f} + \beta_{U} \Big(E(R_{Mkt}) - r_{f} \Big) \Big) - \Big(r_{f} + \beta_{D} \Big(E(R_{Mkt}) - r_{f} \Big) \Big) \Big)$$

Note: adding and subtracting r_f

$$r_E = r_U + \frac{D}{E} (r_U - r_D) \tag{14.5}$$

 \Rightarrow since $r_D < r_U$, r_E increases as increase leverage

2. Leverage and expected return

$$E(R_U) = \left(\frac{E}{E+D}\right)E(R_E) + \left(\frac{D}{E+D}\right)E(R_D)$$

Note: if own all of a firm's equity and debt, have created an unlevered position in the firm

$$E(R_E) = E(R_U) + \left(\frac{D}{E}\right)(E(R_U) - E(R_D))$$

Note: solving for $E(R_E)$

3. Weighted-Average Cost of Capital and Leverage

$$r_{WACC} = \left(\frac{E}{D+E}\right) r_E + \left(\frac{D}{D+E}\right) r_D$$
$$= \left(\frac{E}{D+E}\right) \left(r_U + \frac{D}{E}\left(r_U - r_D\right)\right) + \left(\frac{D}{D+E}\right) r_D$$

Note: substituting 14.5

$$= \left(\frac{E}{D+E}\right) r_U + \left(\frac{E}{D+E}\right) \frac{D}{E} (r_U - r_D) + \left(\frac{D}{D+E}\right) r_D$$

Note: distributive property

$$= \left(\frac{E}{D+E}\right) r_U + \left(\frac{D}{D+E}\right) (r_U - r_D) + \left(\frac{D}{D+E}\right) r_D$$

Note:
$$\left(\frac{E}{D+E}\right)\frac{D}{E} = \left(\frac{D}{D+E}\right)$$

$$= \left(\frac{E}{D+E}\right) r_U + \left(\frac{D}{D+E}\right) r_U - \left(\frac{D}{D+E}\right) r_D + \left(\frac{D}{D+E}\right) r_D$$

Note: distributive property

$$= \left(\frac{D+E}{D+E}\right) r_U = r_U$$

Notes:

1)
$$-\left(\frac{D}{D+E}\right)r_D + \left(\frac{D}{D+E}\right)r_D = 0$$

2)
$$\left(\frac{E}{D+E}\right)r_U + \left(\frac{D}{D+E}\right)r_U = \left(\frac{E+D}{D+E}\right)r_U = \left(\frac{D+E}{D+E}\right)r_U$$

$$3) \left(\frac{D+E}{D+E} \right) = 1$$