Chapter 10: Capital Markets and the Pricing of Risk

Big Picture:

1) To value a project, we need an interest rate to calculate present values
2) The interest rate will depend on the risk of the project
3) To determine this interest rate, we need to be able to:
   a) measure returns
   b) measure risk
   c) figure out the relationship between risk and return

I. Estimating Risk and Return

Basic approaches:

1) make forecasts about the future
2) look at the past and assume future will be like the past.

A. Estimates based on forecasts

Key: need probability distributions for investments
=> probability (p_R) of each possible return (R)

1. Expected return:
=> return expect to earn on average if invest in assets over and over and if the distribution does not change
=> the higher the number, the greater the return you can expect to earn

$$E(R) = \sum_R p_R \times R \qquad (10.1)$$

where:
p_r = probability of return r
R = possible return

2. Variance (Var) and standard deviation (SD):
=> measures how widely the possible returns are distributed
=> the greater the number, the wider the spread of possible returns
=> an asset with no risk has a variance and standard deviation of zero

$$Var(R) = \sum_R p_R \times \left(R - E(R)\right)^2 \qquad (10.2)$$
$$SD(R) = \sqrt{Var(R)} \qquad (10.3)$$

volatility: standard deviation of a return

=> same units of measurement as expected return

Ex. Given the following possible returns on General Electric (GE) and General Mills (GIS) stock, calculate the expected returns and standard deviation of returns on the two stocks?

Economy	Probability	GE	GIS
Boom	.35	.38	.21
Average	.40	.15	.10
Bust	.25	–.14	.01

Expected return:

$E(R_{GE}) = .16 = .35(.38) + .4(.15) + .25(-.14)$
$E(R_{GIS}) = .11 = .35(.21) + .4(.10) + .25(.01)$

Standard deviation:

$Var(R_{GE}) = .35(.38 - .16)^2 + .4(.15 - .16)^2 + .25(-.14 - .16)^2 = .03926$
$StdDev\ (R_{GE}) = \sqrt{.03926} = .20$
$Var(R_{GIS}) = .35(.21 - .11)^2 + .4(.1 - .11)^2 + .25(.01 - .11)^2 = .0006273$
$StdDev\ (R_{GIS}) = \sqrt{.0006273} = .08$

=> can expect a higher return but more uncertainty if invest in GE

B. Estimates based on historical returns

Key assumption: future will be like the past

1. Realized return: $R_{t+1} = \frac{Div_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t}$ (10.4)

Notes:

1) R_{t+1} = return actually earned between t and $t+1$
2) Div_{t+1} = dividend at $t+1$
3) P_t = stock price at t
4) P_{t+1} = stock price at $t+1$
5) $\frac{Div_{t+1}}{P_t}$ =dividend yield
6) $\frac{P_{t+1} - P_t}{P_t}$ =capital gains yield
7) must calculate a return any time a dividend is paid
8) can calculate at any non-dividend date by assuming a dividend of 0

Ex. Assume the following prices and dividends for Apple (AAPL) stock

Date	Dividend	Price	Days
12/31/2020	0	132.69	
2/5/2021	0.205	136.76	36
5/7/2021	0.22	130.21	127
8/6/2021	0.22	146.14	218
11/5/2021	0.22	151.28	309
12/31/2021	0	177.57	365

What was return between 2/5 and 5/7?

$$R_{2/5\text{-}5/7} = -0.0463 = -4.63\% = .0016 + (-.0479) = \frac{.22}{136.76} + \frac{130.21-136.76}{136.76}$$

Q: What does this tell us about Apple?

2. Realized return over longer periods

Key: usually think in terms of annual returns

a. Allow dividend-period returns to compound
=> $1+R_L = (1+R_{S1})\,(1+R_{S2})\,(1+R_{S3})\ldots.$ (10.5)

Note: assumes reinvesting all of dividends so earn return on them

Ex. Calculate the compound annual return over the year given the following returns per period (same data as previous Apple example):

Date	Dividend	Price	Return
12/31/2020	0.0000	132.690002	
2/5/2021	0.2050	136.759995	3.22%
5/7/2021	0.2200	130.210007	-4.63%
8/6/2021	0.2200	146.139999	12.40%
11/5/2021	0.2200	151.279999	3.67%
12/31/2021	0.0000	177.570007	17.38%

$1+R_{year} = 1.3465 = (1.0322)(0.9537)(1.124)(1.0367)(1.1738)$

=> $R_{year} = 34.65\%$

Q: What does this tell us about Apple?

b. Solve for rate that sets PV of inflows equal to PV of outflows => NPV = 0
=> => essentially solving for Internal Rate of Return (IRR)

Notes:

1) this is not in the book and not in homework from the book, but there are some problems on old quizzes and exams
2) no assumption that reinvest dividends
3) outflows = purchase (or beginning) price of security
4) inflows = dividends (or other payments), sales (or ending) price of security

Ex. Calculate the annual return on Apple if assume dividends are not reinvested.

Date	Dividend	Price	Days
12/31/2020	0	132.69	
2/5/2021	0.205	136.76	36
5/7/2021	0.22	130.21	127
8/6/2021	0.22	146.14	218
11/5/2021	0.22	151.28	309
12/31/2021	0	177.57	365

$$NPV = -132.69 + \frac{.205}{(1+r)^{36/365}} + \frac{.22}{(1+r)^{127/365}} + \frac{.22}{(1+r)^{218/365}} + \frac{.22}{(1+r)^{309/365}} + \frac{177.57}{(1+r)^{365/365}} = 0$$

=> Using Excel: r = .3459 = 34.59%

Q: What does this tell us about Apple?

3. Average Annual Returns: $\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$ (10.6)

where:

T = number of historical returns

R_t = return over year t

=> difficult to get your mind wrapped around a list of returns => need to summarize data

=> $\bar{R}$ equals the return would earn on average if invest year after year and distribution does not change

4. Variance and Volatility (Standard Deviation) of Returns:

$$Var(R) = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2 \qquad (10.7)$$

Note: dividing by *T-1* rather than *T* gives unbiased estimator

Volatility = $SD(R) = \sqrt{Var(R)}$

=> gives spread of possible returns
=> the higher the volatility, the more spread out the returns

Ex. Based on the following annual returns on Apple (AAPL) and General Mills (GIS), how did the average annual returns and volatility of Apple compare to those of General Mills?

Year	Apple	GIS
2021	35%	11%
2020	82%	14%
2019	89%	43%
2018	-5%	-32%
2017	48%	-1%
2016	12%	10%

$\bar{R}_{AAPL} = +43.6\ \% = \frac{1}{6}(35 + 82 + 89 - 5 + 48 + 12)$

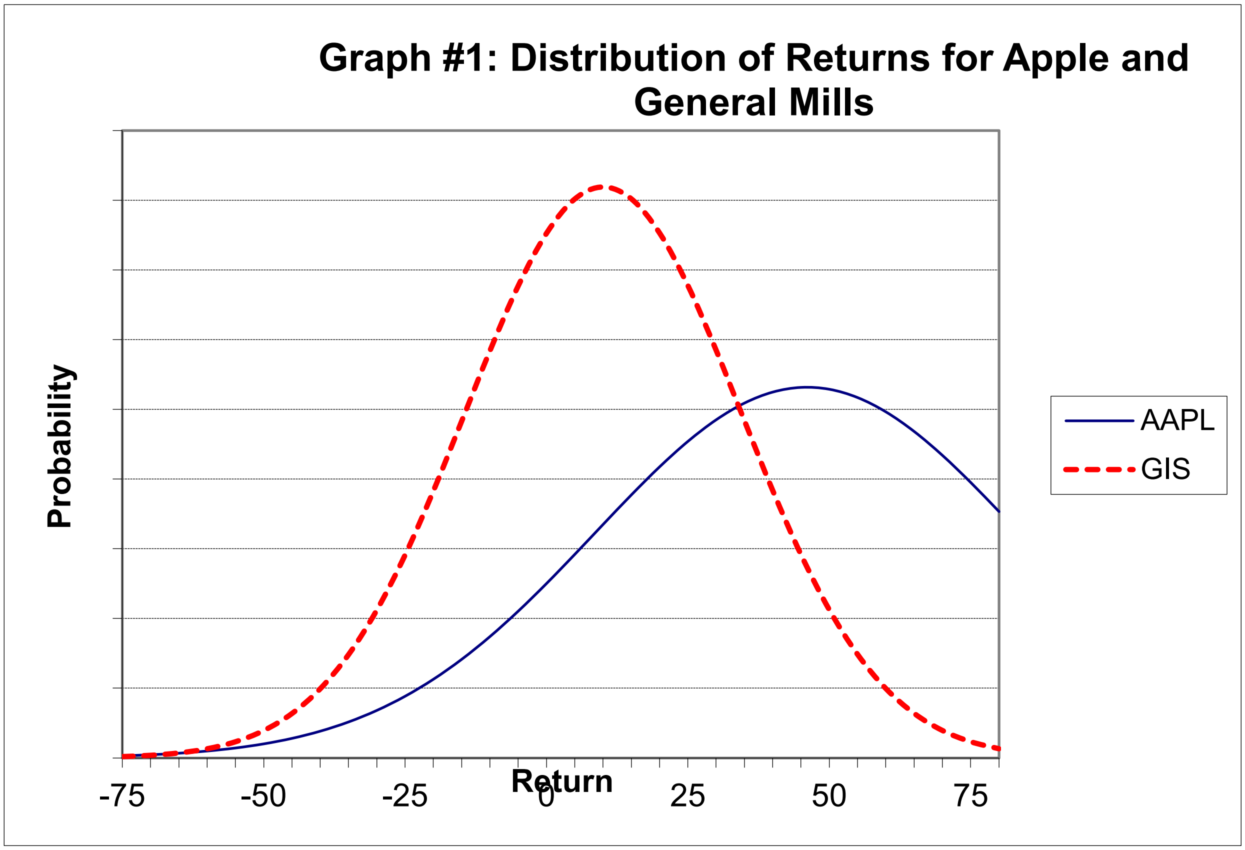
$\bar{R}_{GIS} = +7.5\ \% = \frac{1}{6}(11 + 14 + 43 - 32 - 1 + 10)$

Q: What do these two numbers tell us about Apple and General Mills?

$Var(R_{AAPL}) = 1406 = \frac{1}{5}[(35 - 43.6)^2 + (82 - 43.6)^2 + (89 - 43.6)^2 + (-5 - 43.6)^2 + (48 - 43.6)^2 + (12 - 43.6)^2]$
$SD(R_{AAPL}) = 37.5\% = \sqrt{1406}$

$Var(R_{GIS}) = 590.8 = \frac{1}{5}[(11 - 7.5)^2 + (14 - 7.5)^2 + (43 - 7.5)^2 + (-32 - 7.5)^2 + (-1 - 7.5)^2 + (10 - 7.5)^2]$
$SD(R_{GIS}) = 24.3\% = \sqrt{590.8}$

Q: What do the standard deviations tell us about Apple and General Mills?



Note: To create this graph I assumed that the returns of Apple and General Mills are normally distributed…which is not the case.

Q: Would you invest in Apple or General Mills? Why?

5. Standard Error (SE): Standard Deviation of Average

Notes:

1) the calculated average return is only an estimate of the true average
2) averages vary less than individual observations
3) the bigger our sample, the more confident we are in the average we calculated

=> Need some way to measure uncertainty about our estimate of the average return

Standard Error: $SE = \frac{SD}{\sqrt{N}}$ (10.8)

Where:

SD = standard deviation of the observations (individual returns)
N = number of observations (size of sample)

Ex. Calculate the standard error of returns on Apple in the previous example where the standard deviation of returns over six years equaled 37.5%.

SE (Average return on Apple) $= 15\% = \frac{37.5\%}{\sqrt{6}}$

6. Compound Annual Return

=> annual return required to duplicate the return on an asset over some period

$$CAR = [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_T)]^{1/T} - 1$$

Notes:

1) this is a geometric rather than an arithmetic average
2) the compound annual return is a better description of long-run past performance
3) the average annual return is the best estimate of an investments expected return in the future

Ex. Calculate the compound annual return on Apple and General Mills (GIS) using the data from the previous example.

Year	Apple	GIS
2021	35%	11%
2020	82%	14%
2019	89%	43%
2018	-5%	-32%
2017	48%	-1%
2016	12%	10%

$$CAR(AAPL) = .394 \ = [(1.35)(1.82)(1.89)(0.95)(1.48)(1.12)]^{1/6} - 1$$

=> earning 39.4% per year every year for 6 years would provide the same return as Apple over the 6 years

$$CAR(GIS) = .05 \ = [(1.11)(1.14)(1.43)(0.68)(0.99)(1.10)]^{1/6} - 1$$

=> gaining 5% per year every year for 6 years would have provided the same return as General Mills over the 6 years

II. Information, risk, and return (Review)

1. Types of News and Prices

1) Firm-specific news: good or bad news about company itself
Risk from firm-specific news called: firm-specific, idiosyncratic, unsystematic, unique, diversifiable risk

2) Market-wide news: about economy and thus impacts all stocks
Risk from market-wide news called: systematic, undiversifiable, market risk

2. Risk and Portfolios

1) firm-specific risk diversifies away in large portfolios
2) market risk does not
3) volatility measures the risk of a well-diversified portfolio but not of an individual asset

3. Risk and Return

1) investors will only earn a risk premium for systematic risk

=> no premium for firm-specific risk since diversifies away in a portfolio

2) there is no clear relationship between average returns and volatility (standard deviation)