Chapter 5: Interest Rates

I. Interest Rate Quotes and the Time Value of Money

A. Key ideas

1. Compounding:

2. Interest rates typically quoted in one of two basic ways:

    a. Annual Percentage Rates [APR] –

    b. Effective interest rate [r(t)] –

        t =

        =>

        Ex.

        $r\left(\frac{1}{12}\right)$ = effective monthly rate

        $r(1)$ = effective annual interest rate

        Note: r(1) is also called 1) the APY (Annual Percentage Yield) because of the Truth in Savings Act and 2) the EAR (effective annual rate).

    Ex. Assume given two interest rates for an account. The APR is 6% and the APY is 6.17%.
    => if deposit $100 for a year, end up with $106.17 not $106.

3.

4.

5.

    Ex. monthly cash flows =>

B. Converting interest rates

1. Converting APRs to effective rates

$$r(t) = \frac{APR}{k} \tag{5.2}$$

where:

k =

t = time frame of the interest rate in years = 1/k

Note:

2. Converting between effective interest rates for different time periods

$$r(t) = (1 + r)^n - 1 \tag{5.1}$$

Usefulness: convert to an effective rate that matches the time between cash flows

Notes:

1) n = conversion ratio

2)

3)

Ex. If want an interest rate for a period that is twice as long as the one you start with, n =

Ex. If want an interest rate for a period that is twelve times as long as the one you start with, n =

Ex. If want an interest rate for a period that is one-fourth as long as the one you start with, n =

Ex. Assume an APR of 6% per year with semiannual compounding. What is the effective annual interest rate and the effective monthly interest rate on this account?

$r\left(\frac{1}{2}\right) = .03 =$

=> effective semiannual rate (half a year) is 3%

$r(1) = .0609 =$

$r\left(\frac{1}{12}\right) = .004939 \ =$

Note: $r\left(\frac{1}{2}\right) = .03$, $r(1) = .0609$, and $r\left(\frac{1}{12}\right) = .004939$ are equivalent

=> end up with same amount of money at the end

Ex. If invest $100 for a year, then your account balance at the end of the year equals:

$FV_1 = 106.09 \ =$

Ex. Eight months from today you want to make the first of 12 quarterly withdrawals from a bank account. Your first withdrawal will equal $10,000 and each subsequent withdrawal will grow by 1% each. How much do you need to deposit today if the account pays an APR of 9% with monthly compounding?

Steps: 1) timeline; 2) pattern (annuity); 3) equation; Q: <u>PV</u> or FV? Q: Where end up on timeline?

$r\left(\frac{1}{12}\right) = .0075 =$

$r\left(\frac{1}{4}\right) = .022669 =$

$PV_{5mo} = 109{,}666.07 \ =$

Steps: 2) pattern (single); 3) equation; Q: <u>PV</u> or FV? Q: Where end up on timeline?

$PV_0 = 105{,}644.52 \ =$

Ex. What if you want to make the first withdrawal one month from today (and nothing else changes)?

Q: Will the amount you deposit be larger or smaller than if 1st withdrawal is in eight months?

Steps: 1) timeline; 2) pattern (annuity); 3) equation; Q: PV or FV? Q: Where end up on timeline?

$r\left(\frac{1}{12}\right) = .0075; r\left(\frac{1}{4}\right) = .022669; PV_{-2mo} = 109{,}666.07$

Steps: 2) pattern (single); 3) equation; Q: PV or FV?

$FV_0 = 111{,}317.23 =$

Ex. A bond matures for $1000 three years and ten months from today. The annual coupon on the bond equals $60 but coupons are paid semiannually. What is the value of the bond if it earns a return of 8% per year?

Steps: 1) timeline; 2) pattern (annuity and single); 3) equation; Q: PV or FV? Q: Where end up on timeline?

$r\left(\frac{1}{2}\right) = .03923 =$

Coupons:

$PV_{-2mo} = 202.6257 =$

$FV_0 = 205.2415 =$

Par:

$PV_0 = 744.5187 =$

Price = 949.76= 205.25 + 744.52

Calculator:

$PV_{-2\text{ mo}}$: 30 = PMT, 1000 = FV, 8 = N, 3.923 = I% => PV = 937.6555

FV_0: 937.6555 = PV, 8 = I%, 2/12 = N => FV = 949.76

II. Determinants of interest rates

A. Inflation

Nominal interest rate:

Real interest rate:

Ex. Assume the nominal interest rate is 6% per year and that the real interest rate is 4% per year

=> after one year you will:

1)

2)

1. Basic idea:

2.

=>

3. Converting between nominal and real interest rates

$$r_r = \frac{r - i}{1 + i} \tag{5.5}$$

where:

r = nominal interest rate
i = inflation rate
r_r = real interest rate
Note: can use expected or realized rates

Ex. Assume that the nominal interest rate is 6% per year and that inflation is 5% per year. What is the real interest rate?

r_r = .00952 =

Assume also that you can buy a ton of cocoa for $2200. If you invest the $2200 at 6%, you end up with $2332 in a year, but the cost of a ton of cocoa has risen to $2310. So you can buy 1.0095 tons in a year.

Calculations:

2332 =

2310 =

1.0095 =

Note: the difference between the nominal rate and the inflation rate is a pretty good approximation of the real rate if inflation is low (1% = 6% - 5% in example)

B. The Fed

Basic idea:

Key:

C. Maturity

Basic ideas:

1)

Ex. You can see how interest rates on U.S. Treasuries vary with maturity by googling Treasury rates or by following this link: https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

Note: credit default swap prices now indicate a nonzero chance of default by the U.S. Treasury

2)

3)

D. Taxes

After-tax interest rate: $r_{AT} = r - (\tau \times r) = r(1 - \tau)$ (5.8)

Where:
r_{AT} = after-tax interest rate
r = before-tax interest rate
τ = tax rate

Basic idea:

=>

E. Risk

Basic idea:

Chapter 5 Appendix

A. Discount Rates for a Continuously Compounded APR

Key issue: converting between APR and effective annual interest rate when interest is compounded continuously

Note: Can't use equation 5.2 since k = ∞

$r(1) = e^{APR} - 1$ (5A.1)

APR = ln(1+*r(1)*) (5A.2)

Ex. Assume a bank pays an APR of 5% with continuous compounding. What is the effective annual interest rate?

$r(1) = .05127 =$

Excel: =

B. Continuously Arriving Cash Flows

=> skip this section