Chapter 5: Interest Rates

I. Interest Rate Quotes and the Time Value of Money

A. Key ideas

1. Compounding: **earn interest on interest because interest is added to the balance.**

2. Interest rates typically quoted in one of two basic ways:

   a. Annual Percentage Rates [APR] – **annual interest rate that ignores the impact of compounding**

   b. Effective interest rate [r(t)] – **interest rate that includes the impact of compounding**
   t = **time frame of the interest rate in years**
   => **actual interest rate per period t**

   Ex.
   $r\left(\frac{1}{12}\right)$ = effective monthly rate
   $r(1)$ = effective annual interest rate

   Note: r(1) is also called 1) the APY (Annual Percentage Yield) because of the Truth in Savings Act and 2) the EAR (effective annual rate).

   Ex. Assume given two interest rates for an account. The APR is 6% and the APY is 6.17%.
   => if deposit $100 for a year, end up with $106.17 not $106.

3. **With the exception of continuous compounding (not covered in this class), use only effective interest rates in time value of money calculations**

4. **When calculating the PV or FV of a single cash flow, can use any effective rate**

5. **When calculating the PV or FV of a series of cash flows, must use the effective rate that matches the time between the cash flows.**

   Ex. monthly cash flows => **must use effective monthly rate**

B. Converting interest rates

1. Converting APRs to effective rates

$$r(t) = \frac{APR}{k} \tag{5.2}$$

where:

k = **number of compounding periods per year**
t = time frame of the interest rate in years = 1/k
Note: **any time given an APR, must start with this equation**

2. Converting between effective interest rates for different time periods

$$r(t) = (1 + r)^n - 1 \tag{5.1}$$

Usefulness: convert to an effective rate that matches the time between cash flows

Notes:

1) n = conversion ratio
2) **to convert to a longer period, n > 1**
3) **to convert to a shorter period, n < 1**

Ex. If want an interest rate for a period that is twice as long as the one you start with, n = **2**

Ex. If want an interest rate for a period that is twelve times as long as the one you start with, n = **12**

Ex. If want an interest rate for a period that is one-fourth as long as the one you start with, n = **1/4**

Ex. Assume an APR of 6% per year with semiannual compounding. What is the effective annual interest rate and the effective monthly interest rate on this account?

$r\left(\frac{1}{2}\right) = .03 = \frac{\mathbf{.06}}{\mathbf{2}}$

=> effective semiannual rate (half a year) is 3%

$r(1) = .0609 = (\mathbf{1.03})^{\mathbf{2}} - \mathbf{1}$

$r\left(\frac{1}{12}\right) = .004939 = (\mathbf{1.03})^{\mathbf{1/6}} - \mathbf{1}$

Note: $r\left(\frac{1}{2}\right) = .03$, $r(1) = .0609$, and $r\left(\frac{1}{12}\right) = .004939$ are equivalent

=> end up with same amount of money at the end

Ex. If invest \$100 for a year, then your account balance at the end of the year equals:

$$FV_1 = 106.09 = \mathbf{100}(\mathbf{1.03})^{\mathbf{2}} = \mathbf{100}(\mathbf{1.0609}) = \mathbf{100}(\mathbf{1.004939})^{\mathbf{12}}$$

Ex. Eight months from today you want to make the first of 12 quarterly withdrawals from a bank account. Your first withdrawal will equal \$10,000 and each subsequent withdrawal will grow by 1% each. How much do you need to deposit today if the account pays an APR of 9% with monthly compounding?

Steps: 1) timeline; 2) pattern (annuity); 3) equation; Q: <u>PV</u> or FV? Q: Where end up on timeline?

$r\left(\frac{1}{12}\right) = .0075 = \frac{\mathbf{.09}}{\mathbf{12}}$

$r\left(\frac{1}{4}\right) = .022669 = (\mathbf{1.0075})^{\mathbf{3}} - \mathbf{1}$

$PV_{5mo} = 109{,}666.07 = \frac{\mathbf{10{,}000}}{\mathbf{.022669-.01}}\left(\mathbf{1} - \left(\frac{\mathbf{1.01}}{\mathbf{1.022669}}\right)^{\mathbf{12}}\right)$

Steps: 2) pattern (single); 3) equation; Q: <u>PV</u> or FV? Q: Where end up on timeline?

$PV_0 = 105{,}644.52 = \frac{109{,}666.07}{(1.0075)^5}$

Ex. What if you want to make the first withdrawal one month from today (and nothing else changes)?

Q: Will the amount you deposit be larger or smaller than if 1st withdrawal is in eight months? **Larger**

Steps: 1) timeline; 2) pattern (annuity); 3) equation; Q: PV or FV? Q: Where end up on timeline?

$r\left(\frac{1}{12}\right) = .0075; r\left(\frac{1}{4}\right) = .022669; PV_{-2mo} = 109{,}666.07$

Steps: 2) pattern (single); 3) equation; Q: PV or FV?

$FV_0 = 111{,}317.23 = \mathbf{109{,}666.07(1.0075)^2}$

Ex. A bond matures for $1000 three years and ten months from today. The annual coupon on the bond equals $60 but coupons are paid semiannually. What is the value of the bond if it earns a return of 8% per year?

Steps: 1) timeline; 2) pattern (annuity and single); 3) equation; Q: PV or FV? Q: Where end up on timeline?

$r\left(\frac{1}{2}\right) = .03923 = \mathbf{(1.08)^{1/2} - 1}$

Coupons:

$$PV_{-2mo} = 202.6257 = \frac{\mathbf{30}}{\mathbf{.03923}}\left(\mathbf{1} - \left(\frac{\mathbf{1}}{\mathbf{1.03923}}\right)^{\mathbf{8}}\right)$$
$$FV_0 = 205.2415 = \mathbf{202.6257(1.08)^{2/12}}$$

Par:

$$PV_0 = 744.5187 = \frac{\mathbf{1000}}{\mathbf{(1.08)^{3\frac{10}{12}}}}$$

Price = 949.76= 205.25 + 744.52

Calculator:

$PV_{-2\,mo}$: 30 = PMT, 1000 = FV, 8 = N, 3.923 = I% => PV = 937.6555

FV_0: 937.6555 = PV, 8 = I%, 2/12 = N => FV = 949.76

II. Determinants of interest rates

A. Inflation

Nominal interest rate: **growth rate of money**
Real interest rate: **growth rate of purchasing power**

Ex. Assume the nominal interest rate is 6% per year and that the real interest rate is 4% per year

=> after one year you will:

1) **have 6% more dollars**
2) **be able to buy 4% more stuff**

1. Basic idea: **investors care about real rather than nominal interest rates**

2. **if investors expect inflation to increase, nominal rates will increase**

=> **compensates investors for their loss of purchasing power**

3. Converting between nominal and real interest rates

$$r_r = \frac{r - i}{1 + i} \tag{5.5}$$

where:

r = nominal interest rate
i = inflation rate
r_r = real interest rate
Note: can use expected or realized rates

Ex. Assume that the nominal interest rate is 6% per year and that inflation is 5% per year. What is the real interest rate?

r_r = .00952 = $\frac{.06-.05}{1.05}$

Assume also that you can buy a ton of cocoa for $2200. If you invest the $2200 at 6%, you end up with $2332 in a year, but the cost of a ton of cocoa has risen to $2310. So you can buy 1.0095 tons in a year.

Calculations:

2332 = **2200 (1.06)**
2310 = **2200(1.05)**
1.0095 = **2332/2310 = 1+ r_r**

Note: the difference between the nominal rate and the inflation rate is a pretty good approximation of the real rate if inflation is low (1% = 6% - 5% in example)

B. The Fed

Basic idea: **The Federal Reserve lowers or raises interest rates to stimulate or cool off the economy.**
Key: **if lower (raise) interest rates, more (fewer) investments worthwhile since NPVs rise (fall)**

C. Maturity

Basic ideas:

1) **interest rates vary by maturity**
Ex. You can see how interest rates on U.S. Treasuries vary with maturity by googling Treasury rates or by following this link: https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield
Note: credit default swap prices now indicate a nonzero chance of default by the U.S. Treasury
2) **long-term rates usually exceed short-term rates**
3) **long-term rates reflect what investors expect will happen to short-term rates in the future**

D. Taxes

After-tax interest rate: $r_{AT} = r - (\tau \times r) = r(1 - \tau)$ (5.8)

Where:
r_{AT} = after-tax interest rate
r = before-tax interest rate
τ = tax rate

Basic idea: **investors care about after-tax returns**

=> **the higher the tax rates, the higher the return investors will demand**

E. Risk

Basic idea: **the greater the risk, the higher the interest rate**

Chapter 5 Appendix

A. Discount Rates for a Continuously Compounded APR

Key issue: converting between APR and effective annual interest rate when interest is compounded continuously

Note: Can't use equation 5.2 since k = ∞

$r(1) = e^{APR} - 1$ (5A.1)

APR = ln(1+$r(1)$) (5A.2)

Ex. Assume a bank pays an APR of 5% with continuous compounding. What is the effective annual interest rate?

$r(1) = .05127 \ = \boldsymbol{e^{.05} - 1}$

Excel: = **exp(.05) – 1**

B. Continuously Arriving Cash Flows

=> skip this section