

## Chapter 3: Arbitrage and Financial Decision Making

Fundamental question: What are assets worth?

=> starting point: any two assets that always pay the same cash flows should have the same price

=> if not the case, whoever notices it can make a lot of money very quickly.

=> all mispriced assets will disappear almost immediately as bought (if price too low) or sold (if price too high)

### I. Financial Decision Making

#### A. Steps

##### 1. Identify costs and benefits

Note: work with accountants, managers, economists, lawyers, etc. to determine costs and benefits

##### 2. Convert costs and benefits to equivalent dollars today

##### 3. Proceed if the value of the benefits exceed the value of the costs

#### B. Equivalent Dollars (Value) Today

##### 1. When competitive markets exist

###### a. Definitions and example

Competitive market: **goods can be bought and sold at the same price**

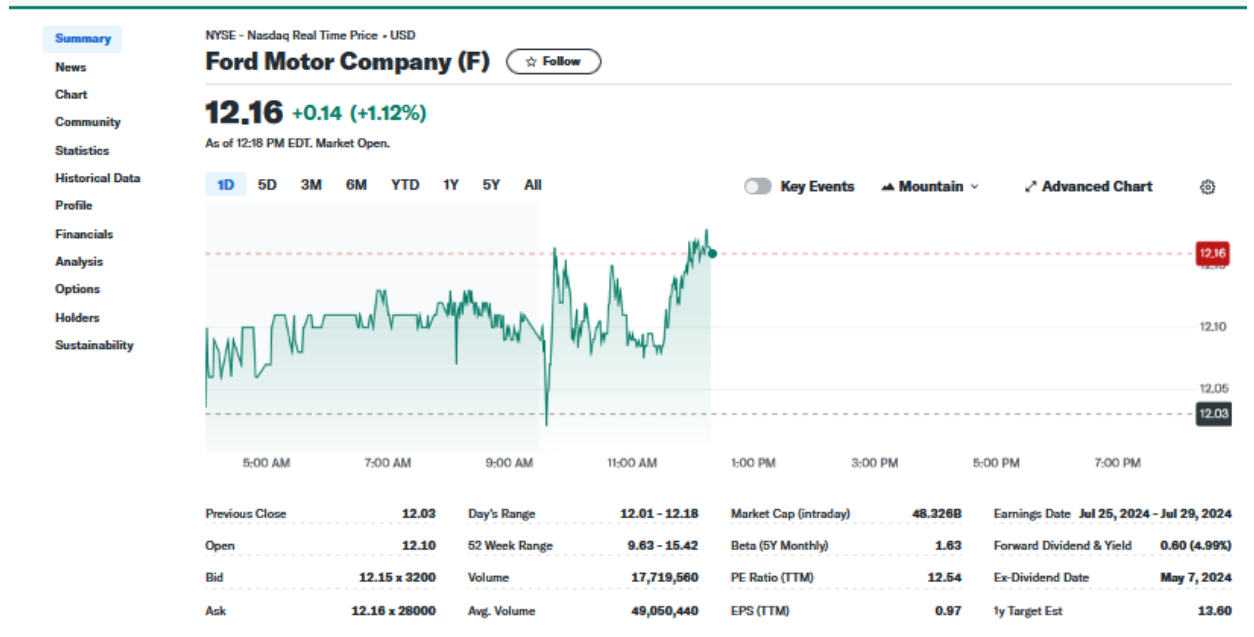
Q: Do such markets exist?

=> **The NYSE is pretty close**

Bid price: **highest price at which anyone is willing to buy**

Ask price: **lowest price at which anyone is willing to sell**

=> Ford Example:



Current Ford quote: <https://finance.yahoo.com/quote/F>

Notes:

- 1) anyone can submit their own bid or ask price  
=> called a limit order
- 2) anyone submitting a market order takes whatever price exists in the market now  
=> if buying, they'll pay whatever the ask price is (the lowest price that anyone is willing to sell for)  
=> if selling, they'll get whatever the bid price is (the highest price that anyone is willing to pay).

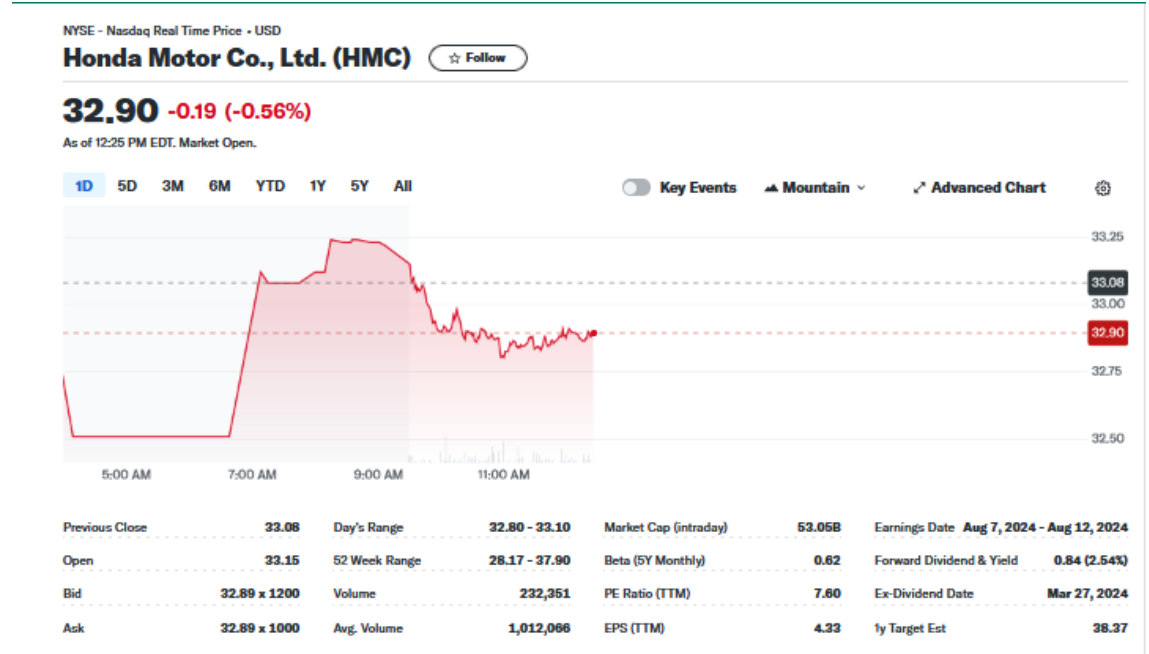
b. Equivalent value today if competitive market: **market price**

Note: **value doesn't depend on individual preferences or expectations**

Ex. Assume your uncle gives your 100 shares of Ford. What is the gift worth?

Ex. Would you trade your shares for \$1000?

Ex. Would you trade your shares for 100 shares of Honda?



Current Honda Quote: <https://finance.yahoo.com/quote/HMC>

2. When a competitive market does not exist

Note: This is when finance gets more interesting

Ex. Ford dealership on Hwy 84 in Waco (Bird Kultgen)

a. Equivalent value today: **present value of future cash flows**

Notes:

- 1) **cash flows at different points in time are in different units**  
=> **can't combine or compare them**
- 2) **interest rate: exchange rate across time**  
=> **allows us to convert dollar at one point in time to another point in time**
- 3) **interpretation: present value = amount would need to invest today at the current interest rate to end up with the same cash flow in the future**

## C. Making Decisions

- 1) Accept all positive NPV projects or the highest NPV project if must chose
- 2) NPV = present value of all cash flows (inflows and outflows)
- 3) Interpretation of NPV: **wealth created by project**
- 4) Another way to think about it: **NPV equals the difference between the cost of the project and how much it would cost to recreate a project's cash flows at the current interest rate**
- 5) Decision doesn't depend on preference for cash today vs. cash in the future

Ex. Assume you have an opportunity to buy land for \$110,000 that you will be able to sell for \$120,000 a year from today. Further assume that all cash flows are known for sure and that you can borrow or lend at a risk-free interest rate of 4%.

a. Should you buy the land if you have \$110,000?

**NPV =**

Q: How much would you have to invest at 4% to end up with \$120,000 a year from today?

**=> \$115,384.60**

Q: How much better off are you if you buy the land? **\$5384.60**

**=> \$5384.60 equals the difference between the \$100,000 cost to buy the land and the 115,384.60 it would take to recreate the \$120,000 a year from today through the financial markets.**

b. Should you buy the land if you have no money?

=> yes

*Q: How?*

=> **borrow \$115,384.60 and buy the land for \$110,000**

=> **keep \$5384.60 today**

=> **in one year sell the land and use the proceeds to pay off the loan**

*Q: Is it realistic to assume that a firm or individual could borrow more than a project costs and to keep the difference?*

*A: Not really. Rules implemented after the global financial crisis of 2007 – 2008 prevent borrowing more than the value of an asset. Before the financial crisis, some people borrowed more than the value of a house, but most of that involved fraud. But the idea is theoretically sound.*

## II. Arbitrage and the Law of One Price

### A. Introduction and Definitions

1. Arbitrage: **trading to take advantage of price differences between equivalent assets possibly trading in different markets**

Notes:

- 1) key transactions: **buy low-priced asset and simultaneously sell the equivalent high-priced-asset**
- 2) equivalent assets: **assets with exactly the same cash flows in all periods under all conditions**
- 3) **arbitrage requires no investment and creates a riskless payoff today**

Ex. Assume that the price for GE stock is \$160 on the New York Stock Exchange and \$150 on the NASDAQ.

Arbitrage: **simultaneously buy a share on the NASDAQ and sell a share on the NYSE.**

Arbitrage profit: **\$10 today with no risk and no investment**

*Q: How many shares want to simultaneously buy and sell?*

Ex. Assume the following prices for HMC stock are available on the CBOE and the New York Stock exchange:

Note: HMC stock traded on CBOE and NYSE are equivalent since same exact asset

CBOE				NYSE			
Bid		Ask		Bid		Ask	
<u>Price</u>	<u>Size</u>	<u>Price</u>	<u>Size</u>	<u>Price</u>	<u>Size</u>	<u>Price</u>	<u>Size</u>
25.73	7000	25.76	6000	25.88	26,000	25.89	35,000

Q: What transactions create arbitrage? What is the profit?

Arbitrage: **simultaneously buy shares on the CBOE and sell shares on the NYSE for risk-free profit of \$720.**

Note: Arbitrage profit =

Q: Why do we use \$25.88 and \$25.76?

Q: Why not trade more than 6000 shares?

Q: How long will these conditions last?

=> exploiting arbitrage eliminates arbitrage opportunities

Current Quotes:

CBOE: [https://www.cboe.com/us/equities/market\\_statistics/book/hmc/](https://www.cboe.com/us/equities/market_statistics/book/hmc/)

NYSE (on Yahoo): <https://finance.yahoo.com/quote/HMC>

2. Normal market: **no arbitrage possible**

Reason should be “normal”: **arbitrage will only exist until someone notices it...and a lot of people are looking for such opportunities.**

3. Equivalent assets: **assets with exactly the same cash flows**

4. Law of one price: **equivalent assets trading at the same time in different normal markets must have the same price**

## 5. Short sales:

- 1) today: **borrow a security (usually from a broker) and sell it**
- 2) later: **buy same security and give it back to whoever you borrowed it from**

Notes:

- 1) **if the security has matured, might pay the cash value rather than buying the security and giving it back**
- 2) **must make up any cash flows the lender would have received while the security was borrowed**
- 3) **short seller can buy and return the security at any time**
- 4) **lender can demand the return of the loaned security at any time**

Ex. Assume you want to short-sell 100 shares of GE today for the market price of \$160 per share

- 1) **borrow 100 shares from your broker and sell them on the NYSE**

*Q: Where stand?*

=> **owe your broker 100 shares of GE**

=> **have \$16,000 (160 x 100) in your brokerage account**

- 2) assume price falls from \$160 to \$155

- 3) Q: How close out short position?

=> **buy 100 shares at \$155 per share and give the shares to your broker**

- 4) assume that while you were short GE paid a dividend of \$0.28 per share

=> **must give \$28 to your broker.**

- 5) Profit = \$972 =

## B. No Arbitrage Prices for Securities

**Key: For there to be no arbitrage, the price of any security must equal the present value of its cash flows**

Ex. Assume you can borrow or deposit in a bank at the risk-free rate of 7% and that a risk-free bond pays \$1000 a year from today

$PV =$

Goal in arbitrage: positive cash flow today, no possible net cash flow after today

Basic questions to ask when setting up an arbitrage:

- 1) What transaction (or set of transactions) is equivalent to the security?
- 2) Do you want to buy or sell the security?
- 3) What cash flows does this create?
- 4) What transaction today offsets the security's cash flows in the future?

Q: What are equivalent transactions?

<u>Transaction</u>	<u>Equivalent Transaction</u>	<u>\$ today</u>	<u>\$ in one year</u>
Buy bond			

Short-sell bond

- a) Assume price of bond is \$920 (rather than its present value)  
=> *arbitrage is possible*

Q: Buy or sell the bond if the price is \$920 rather than \$934.58?

**Buy (buy low and sell high).**

Q: What cash flows does this create?

**Today = -\$934.58; One year = +\$1000**

Q: What transaction today offsets the security's cash flows in the future?

**Borrow \$934.58**

Table solution:

<u>Transaction(t=0)</u>	<u>\$ today</u>	<u>\$ in one year</u>	<u>Transaction(t=1)</u>
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Total

Arbitrage profit =



b) Assume price of bond is \$950 (rather than its present value)

Q: Buy or sell the bond if the price is \$950 rather than \$934.58?

**Sell (buy low and sell high).**

Q: What cash flows does this create?

**Today = +\$934.58; One year = -\$1000**

Q: What transaction today offsets the security's cash flows in the future?

**Deposit \$934.58**

Table solution:

<u>Transaction (t=0)</u>	<u>\$ today</u>	<u>\$ in one year</u>	<u>Transaction (t=1)</u>
<hr/>			
Total			

Arbitrage profit = **\$15.42**

=> only way there is no arbitrage: **price = \$934.58**

Notes:

- 1) investors rushing to take advantage of any arbitrage opportunity will quickly drive the price to \$934.58
- 2) interest rates are usually extracted from security prices rather than the other way around

Ex. What is usually known:  $CF_1 = \$1000$ , Price = \$934.58

$$\Rightarrow 934.58 = \frac{1000}{1+r} \Rightarrow r = .07 = 7\%$$

- 3) In a normal market, buying and selling securities has zero NPV

Keys:

a) NPV(buying security) = **PV(CF) - price**

=> in normal market, price = PV(CF)

b) NPV(selling security) = **price - PV(CF)**

=> in normal market, price = PV(CF)

c) otherwise arbitrage possible

C. No Arbitrage Prices of Portfolios

Portfolio: collection of securities

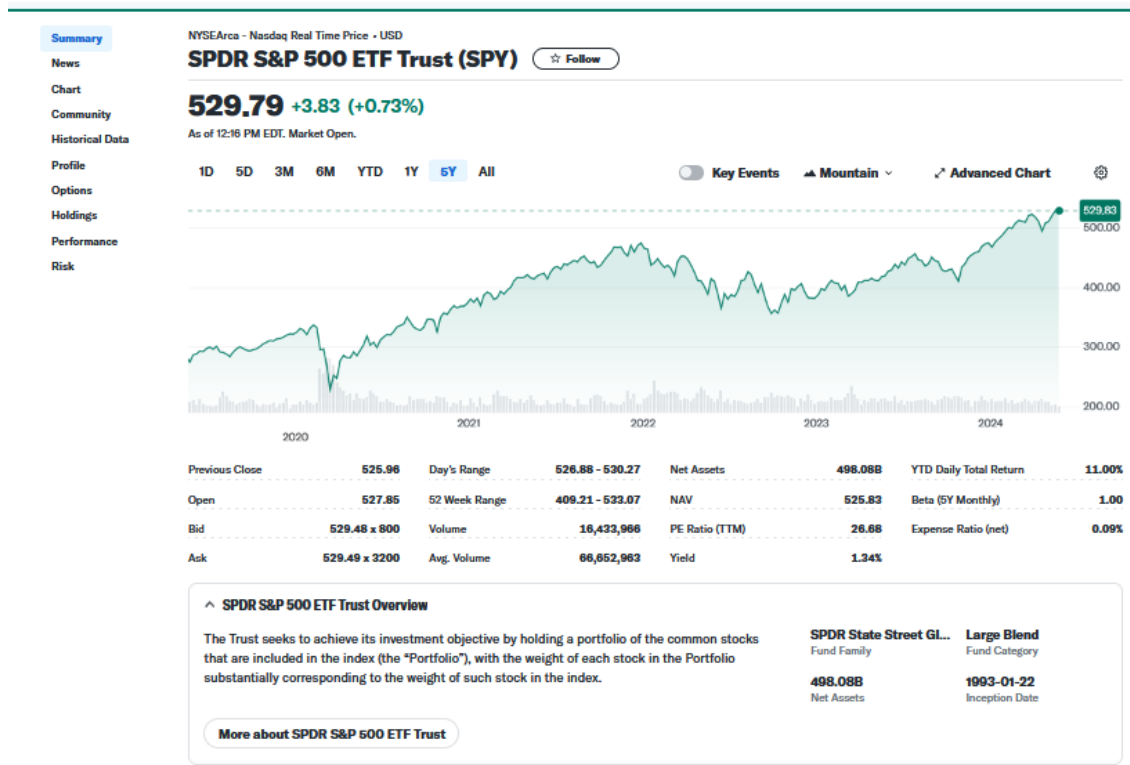
**Key: In a normal market, equivalent portfolios (exactly same cash flows) must have same price**

=> otherwise arbitrage is possible

1. ETF: exchange traded fund

=> **essentially a portfolio of securities that you can trade on an exchange**

Ex. SPDR S&P500 ETF Trust



Current Quotes:

SPY: <https://finance.yahoo.com/quote/SPY>

S&P500: <https://finance.yahoo.com/quote/%5EGSPC>

2. Value Additivity: the price of a portfolio must equal the combined values of the securities in the portfolio

$$\Rightarrow \text{Price}(A+B) = \text{Price}(A) + \text{Price}(B) \quad (3.5)$$

Ex. Assume the following:

ETF1 has one share of security A and one share of security B.

ETF2 has one share of security C and one share of security D.

Security A pays \$100 a year from today and has a market price of \$95.24.

Security B pays \$150 a year from today and has a market price of \$142.86.

Security C pays \$200 a year from today and Security D pays \$50 a year from today.

Q: What portfolio is equivalent to ETF1?

<u>Transaction</u>	<u>\$ in one year</u>
Buy ETF1	+250.00

Equivalent portfolio:

Buy A	+100.00
<u>Buy B</u>	<u>+150.00</u>
Total	+250.00

$\Rightarrow$  **buying a share of A and a share of B is equivalent to buying the ETF**

Q: What is the no-arbitrage price be for ETF1?

$$\Rightarrow 238.10 = 95.24 + 142.86$$

Reason: **ETF1 must have the same price as a portfolio of A and B**

Key to arbitrage with equivalent portfolios with different prices: **buy low and sell high**

Assume price of ETF1 is \$220 instead of \$238.10

***Arbitrage: Buy ETF1, short-sell equivalent portfolio***

<u>Transaction (t=0)</u>	<u>\$ today</u>	<u>\$ in one year</u>	<u>Transaction (t=1)</u>
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<u>Total</u>	<u>+</u>	<u>0.00</u>
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Assume price of ETF1 is \$245

*Arbitrage: short-sell ETF1, buy equivalent portfolio*

Transaction (t=0)    \$ today    \$ in one year    Transaction (t=1)

<b>Total</b>	+	<b>0.00</b>
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=> only way no arbitrage: price of ETF1 = 238.10  
=> arbitrage will quickly drive the price of ETF1 to \$238.10

Q: What does the market price for ETF2 have to be?

Note: payoff on ETF2 next year:  $200 + 50 = 250$

Q: What portfolio is equivalent to ETF2?

=> **ETF1**

=> **must be worth 238.10**

Reason: **otherwise arbitrage is possible between ETF1 and ETF2**

## 2. Value Additivity and Firm Value

Key issues:

=> value of firm = sum of value of individual assets  
=> change in value of firm from decision = NPV of decision

## III. Appendix to Chapter 3: The Price of Risk

### A. Risky Verses Risk-Free Cash Flows

#### 1. Key ideas

##### 1) investors prefer less risk other things equal

Reason: for most people a \$1 loss is a bigger deal than a \$1 gain

##### 2) Risk premium: extra return demanded by investors for holding risky assets instead of Treasuries

=> compensates investors for taking any risk

## 2. Risk premium on the market

=> **additional return can expect for taking the market's risk**

Note: the market risk premium will increase if:

- **the risk of the market increases or,**
- **if investors become more risk averse**

## 3. Risk premium on a security

Key => Depends on two things:

- 1) **risk premium on market index**
- 2) **degree to which security's return varies with market index.**

=> **more varies with market, higher the risk premium**

Ex. Assume the following:

- risk-free interest rate = 2%
- a strong or weak economy is equally likely
- price of the market index: \$100
- payoff on stock market index depends on the economy as follows:
  - weak economy = \$75
  - strong economy = \$139
- payoff on Orange Inc. depends on the economy as follows:
  - weak economy = \$95
  - strong economy = \$159

Q: What are the expected cash flow next year, the possible returns, the expected return, and the risk premium on the market?

=> expected cash flow for the market index = 107 =

=> return on the market depends on the economy as follows:

Strong: 39% =

Weak: 0 - 25% =

=> expected return on the market index: 7% =

=> risk premium on the market index = 5% =

Q: What is the no-arbitrage price of Orange Inc.?

*Q: How does the payoff on Orange compare to the payoff on the market?*

**=> Orange always pays \$20 more than the market**

*Q: How create a portfolio that is equivalent to Orange?*

<u>Transaction</u>	\$ in one year	
	<u>Weak</u>	<u>Strong</u>
Buy Orange		

Equivalent Portfolio:

\_\_\_\_\_

Total

Cost to build portfolio that is equivalent to Orange:

**=> Cost of equivalent portfolio = 119.61 =**

**=> the price of Orange must equal 119.61 => otherwise arbitrage**

Q: What is arbitrage profit if the price of Orange is \$125 instead of \$119.61? How do you create this profit?

<u>Transaction</u>	<u>\$ today</u>	\$ in one year		<u>Transaction</u>
		<u>Weak</u>	<u>Strong</u>	
_____	_____	_____	_____	
Total	+	<b>0.00</b>	<b>0.00</b>	

Q: What is the arbitrage profit if the price of Orange is \$110 instead of \$119.61?

<u>Transaction</u>	<u>\$ today</u>	\$ in one year		<u>Transaction</u>
		<u>Weak</u>	<u>Strong</u>	
_____	_____	_____	_____	
Total	+	<b>0.00</b>	<b>0.00</b>	

Q: What are the possible returns, expected return, and risk premium on Orange if it is correctly priced at \$119.61?

Return on Orange if strong economy = 32.9% =

Return on Orange if weak economy = -20.6% =

Note: **return on Orange less volatile than the market (+39% or -25%)**

Q: How should the risk premium on Orange compare to the market (5%)?

=> **should be less**

Expected cash flow for Orange = 127=

Expected return on Orange = .062 = 6.2% =

Risk premium on Orange = .042 =



Ex. Assume that all of the information in the Orange example still holds (Market index trades for \$100 today and pays, \$75 or \$139 a year from today. Risk-free rate equals 2%). Assume also that we can invest in Pineapple which pays \$65 when the economy is weak and \$129 when the economy is strong?

Q1: What is the no-arbitrage price for Pineapple?

Q2: What is the arbitrage profit if Pineapple's price is \$95 or \$80?

Q3: If Pineapple is correctly priced, what are the possible returns, expected return, and risk premium on the stock?

Note: **Pineapple always pays \$10 less than the market.**

Equivalent portfolio:

<u>Transaction</u>	\$ in one year	
	<u>Weak</u>	<u>Strong</u>
Buy Pineapple		

Equivalent Portfolio:

	_____	_____
Total		

$$\text{Cost of equivalent portfolio} = 90.20 = 100 - \frac{10}{1.02} = 100 - 9.80$$

A1: no-arbitrage price of Pineapple = \$90.20

A2 (\$95): Arbitrage profit if the price of Pineapple is \$95 instead of the no-arbitrage price of \$90.20.

<u>Transaction</u>	<u>\$ today</u>	\$ in one year		<u>Transaction</u>
		<u>Weak</u>	<u>Strong</u>	
Buy Pineapple				
Buy Market Index				
Total	+	0.00	0.00	

A2 (\$80): Arbitrage profit if the price of Pineapple is \$80 instead of the no-arbitrage price of \$90.20.

<u>Transaction</u>	<u>\$ today</u>	<u>\$ in one year</u>		<u>Transaction</u>
		<u>Weak</u>	<u>Strong</u>	
Total	+	<u>0.00</u>	<u>0.00</u>	

A3: Possible returns, expected return, and risk premium on Pineapple if it is correctly priced at \$90.20

Return on Pineapple if strong economy = 43% =

Return on Pineapple if weak economy = -27.9% =

Note: return on Pineapple is more volatile than the market (+39% or -25%)

Expected return on Pineapple = .0755 = 7.55%

Risk premium on Pineapple = .0555 =

Note: Risk premium on Pineapple larger than 5% on market.

E. Transaction cost: cost to trade securities

Note: transaction costs include:

1. commission to broker
2. bid-ask spread: difference between bid price and ask price

Key: Transaction costs lead to the following modifications of earlier definitions:

Normal market => no arbitrage after transaction costs covered

Law of one price => difference in prices for equivalent securities must be less than transaction costs

No arbitrage price => differences between price and the PV(CF) must be less than transaction costs

Portfolio prices => Difference between the price of a portfolio and the sum of the prices of assets in the portfolio must be less than the transaction costs to build or break apart the portfolio