## Chapter 3: Arbitrage and Financial Decision Making

Fundamental question: What are assets worth?
$=>$ starting point: any two assets that always pay the same cash flows should have the same price
$=>$ if not the case, whoever notices it can make a lot of money very quickly.
$=>$ all mispriced assets will disappear almost immediately as bought (if price too low) or sold (if price too high)
I. Financial Decision Making
A. Steps

1. Identify costs and benefits

Note: work with accountants, managers, economists, lawyers, etc. to determine costs and benefits
2. Convert costs and benefits to equivalent dollars today
3. Proceed if the value of the benefits exceed the value of the costs
B. Equivalent Dollars (Value) Today

1. When competitive markets exist
a. Definitions and example

Competitive market:
Q: Do such markets exist?

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=>
$$

Bid price:
Ask price:
=> Ford Example:


Current Ford quote: https://finance.yahoo.com/quote/F
Notes:

1) anyone can submit their own bid or ask price
=> called a limit order
2) anyone submitting a market order takes whatever price exists in the market now
=> if buying, they'll pay whatever the ask price is (the lowest price that anyone is willing to sell for)
$=>$ if selling, they'll get whatever the bid price is (the highest price that anyone is willing to pay).
b. Equivalent value today if competitive market:

Note:

Ex. Assume your uncle gives your 100 shares of Ford. What is the gift worth?
Ex. Would you trade your shares for $\$ 1000$ ?

Ex. Would you trade your shares for 100 shares of Honda?


Current Honda Quote: https://finance.yahoo.com/quote/HMC
2. When a competitive market does not exist

Note: This is when finance gets more interesting
Ex. Ford dealership on Hwy 84 in Waco (Bird Kultgen)
a. Equivalent value today:

Notes:
1)
=>
2) interest rate:
=>
3) interpretation:
C. Making Decisions

1) Accept all positive NPV projects or the highest NPV project if must chose
2) $\mathrm{NPV}=$ present value of all cash flows (inflows and outflows)
3) Interpretation of NPV:
4) Another way to think about it:
5) Decision doesn't depend on preference for cash today vs. cash in the future

Ex. Assume you have an opportunity to buy land for $\$ 110,000$ that you will be able to sell for $\$ 120,000$ a year from today. Further assume that all cash flows are known for sure and that you can borrow or lend at a risk-free interest rate of $4 \%$.
a. Should you buy the land if you have $\$ 110,000$ ?
$\mathrm{NPV}=$

Q: How much would you have to invest at $4 \%$ to end up with $\$ 120,000$ a year from today?
=>

Q: How much better off are you if you buy the land?
$=>\$ 5384.60$ equals the
b. Should you buy the land if you have no money?

$$
=>\text { yes }
$$

Q: How?
=>

$$
\begin{aligned}
& => \\
& =>
\end{aligned}
$$

Q : Is it realistic to assume that a firm or individual could borrow more than a project costs and to keep the difference?

A: Not really. Rules implemented after the global financial crisis of 2007 2008 prevent borrowing more than the value of an asset. Before the financial crisis, some people borrowed more than the value of a house, but most of that involved fraud. But the idea is theoretically sound.
II. Arbitrage and the Law of One Price
A. Introduction and Definitions

1. Arbitrage:

Notes:

1) key transactions:
2) equivalent assets:
3) 

Ex. Assume that the price for GE stock is $\$ 160$ on the New York Stock Exchange and $\$ 150$ on the NASDAQ.

Arbitrage:

Arbitrage profit:
Q: How many shares want to simultaneously buy and sell?

Ex. Assume the following prices for HMC stock are available on the CBOE and the New York Stock exchange:

Note: HMC stock traded on CBOE and NYSE are equivalent since same exact asset

| CBOE |  |  |  | NYSE |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bid | Ask |  |  | Bid | Ask |  |  |
| Price | Size | Price | Size | Price | Size | Price | Size |
| 25.73 | 7000 | 25.76 | 6000 | 25.88 | 26,000 | 25.89 | 35,000 |

Q: What transactions create arbitrage? What is the profit?
Arbitrage:

Note: Arbitrage profit =
Q: Why do we use $\$ 25.88$ and $\$ 25.76$ ?
Q: Why not trade more than 6000 shares?
Q: How long will these conditions last?
=> exploiting arbitrage eliminates arbitrage opportunities

## Current Quotes:

CBOE: https://www.cboe.com/us/equities/market statistics/book/hmc/ NYSE (on Yahoo): https://finance.yahoo.com/quote/HMC
2. Normal market:

Reason should be "normal":
3. Equivalent assets:
4. Law of one price:
5. Short sales:

1) today:
2) later:

Notes:
1)
2)
3)
4)

Ex. Assume you want to short-sell 100 shares of GE today for the market price of $\$ 160$ per share
1)

$$
\begin{aligned}
& \text { Q: Where stand? } \\
& => \\
& =>
\end{aligned}
$$

2) assume price falls from $\$ 160$ to $\$ 155$
3) Q: How close out short position?

$$
=>
$$

4) assume that while you were short GE paid a dividend of $\$ 0.28$ per share =>
5) Profit $=\$ 972=$
B. No Arbitrage Prices for Securities

Key:

Ex. Assume you can borrow or deposit in a bank at the risk-free rate of $7 \%$ and that a risk-free bond pays $\$ 1000$ a year from today
$P V=$

Goal in arbitrage: positive cash flow today, no possible net cash flow after today
Basic questions to ask when setting up an arbitrage:

1) What transaction (or set of transactions) is equivalent to the security?
2) Do you want to buy or sell the security?
3) What cash flows does this create?
4) What transaction today offsets the security's cash flows in the future?

Q: What are equivalent transactions?
Transaction Equivalent Transaction \$ today \$ in one year
Buy bond
Short-sell bond
a) Assume price of bond is $\$ 920$ (rather than its present value)
=> arbitrage is possible
Q: Buy or sell the bond if the price is $\$ 920$ rather than $\$ 934.58$ ?
Q: What cash flows does this create?
Q: What transaction today offsets the security's cash flows in the future?

Table solution:
Transaction $(\mathrm{t}=0) \quad \$$ today $\$$ in one year $\quad$ Transaction $(\mathrm{t}=1)$

## Total

Arbitrage profit $=$
b) Assume price of bond is $\$ 950$ (rather than its present value)

Q : Buy or sell the bond if the price is $\$ 950$ rather than $\$ 934.58$ ?

Q: What cash flows does this create?
Q: What transaction today offsets the security's cash flows in the future?

Table solution:
Transaction ( $\mathrm{t}=0$ ) \$ today \$ in one year Transaction ( $\mathrm{t}=1$ )

## Total

Arbitrage profit $=$
=> only way there is no arbitrage:

Notes:

1) investors rushing to take advantage of any arbitrage opportunity will quickly drive the price to $\$ 934.58$

2 ) interest rates are usually extracted from security prices rather than the other way around

Ex. What is usually known: $\mathrm{CF}_{1}=\$ 1000$, Price $=\$ 934.58$

$$
\Rightarrow 934.58=\frac{1000}{1+r}=>\mathrm{r}=.07=7 \%
$$

3) In a normal market, buying and selling securities has zero NPV

Keys:
a) $\operatorname{NPV}($ buying security $)=$
$=>$ in normal market, price $=\mathrm{PV}(\mathrm{CF})$
b) $\operatorname{NPV}($ selling security $)=$
$=>$ in normal market, price $=\mathrm{PV}(\mathrm{CF})$
c) otherwise arbitrage possible
C. No Arbitrage Prices of Portfolios

Portfolio: collection of securities

Key:
$=>$ otherwise arbitrage is possible

1. ETF: exchange traded fund
=>

## Ex. SPDR S\&P500 ETF Trust



## Current Quotes:

SPY: https://finance.yahoo.com/quote/SPY
S\&P500: https://finance.yahoo.com/quote/\^GSPC
2. Value Additivity: the price of a portfolio must equal the combined values of the securities in the portfolio
$\Rightarrow \operatorname{Price}(A+B)=\operatorname{Price}(A)+\operatorname{Price}(B)$
Ex. Assume the following:
ETF1 has one share of security A and one share of security B.
ETF2 has one share of security $C$ and one share of security D.
Security A pays $\$ 100$ a year from today and has a market price of \$95.24.
Security B pays $\$ 150$ a year from today and has a market price of $\$ 142.86$.
Security C pays $\$ 200$ a year from today and Security D pays $\$ 50$ a year from today.
Q: What portfolio is equivalent to ETF1?
$\frac{\text { Transaction }}{\text { Buy ETF1 }} \quad \frac{\text { \$in one year }}{+250.00}$

Equivalent portfolio:

| Buy A | +100.00 |
| :---: | :--- |
| $\underline{\text { Buy B }}$ | $+\underline{150.00}$ |
| Total | +250.00 |
|  |  |

Q: What is the no-arbitrage price be for ETF1?

$$
=>
$$

## Reason:

Key to arbitrage with equivalent portfolios with different prices:

Assume price of ETF1 is $\$ 220$ instead of $\$ 238.10$
Arbitrage: Buy ETF1, short-sell equivalent portfolio
Transaction ( $\mathrm{t}=0$ ) \$ today \$ in one year Transaction $(\mathrm{t}=1)$
$\overline{\text { Total }} \quad \overline{0.00}$

Assume price of ETF 1 is $\$ 245$
Arbitrage: short-sell ETF1, buy equivalent portfolio
Transaction ( $\mathrm{t}=0$ ) \$ today \$ in one year Transaction ( $\mathrm{t}=1$ )
$\overline{\text { Total }} \quad \overline{\mathbf{0 . 0 0}}$
$\Rightarrow$ only way no arbitrage: price of ETF1 $=238.10$
$=>$ arbitrage will quickly drive the price of ETF1 to $\$ 238.10$

Q: What does the market price for ETF2 have to be?
Note: payoff on ETF2 next year: $200+50=250$
Q: What portfolio is equivalent to ETF2?
=>

$$
=>
$$

Reason:
2. Value Additivity and Firm Value

Key issues:
=> value of firm = sum of value of individual assets
$=>$ change in value of firm from decision $=$ NPV of decision
III. Appendix to Chapter 3: The Price of Risk
A. Risky Verses Risk-Free Cash Flows

1. Key ideas
1) 

Reason: for most people a $\$ 1$ loss is a bigger deal than a $\$ 1$ gain
2) Risk premium: extra return demanded by investors for holding risky assets instead of Treasuries
=> compensates investors for taking any risk
2. Risk premium on the market
=>

Note: the market risk premium will increase if:
3. Risk premium on a security

Key $=>$ Depends on two things:
1)
2)
=>

Ex. Assume the following:

- risk-free interest rate $=2 \%$
- a strong or weak economy is equally likely
- price of the market index: $\$ 100$
- payoff on stock market index depends on the economy as follows:
weak economy $=\$ 75$
strong economy $=\$ 139$
- payoff on Orange Inc. depends on the economy as follows:
weak economy = \$95
strong economy $=\$ 159$
Q: What are the expected cash flow next year, the possible returns, the expected return, and the risk premium on the market?
$=>$ expected cash flow for the market index $=107=$
$=>$ return on the market depends on the economy as follows:
Strong: $39 \%=$
Weak: $0-25 \%=$
=> expected return on the market index: $7 \%=$
$\Rightarrow>$ risk premium on the market index $=5 \%=$

Q: What is the no-arbitrage price of Orange Inc.?
Q: How does the payoff on Orange compare to the payoff on the market?
=>

Q: How create a portfolio that is equivalent to Orange?

|  | \$ in one year <br> Transaction <br> Weak$\underline{\text { Strong }}$ |
| :---: | :---: |

Buy Orange
Equivalent Portfolio:

Total
Cost to build portfolio that is equivalent to Orange:
$=>$ Cost of equivalent portfolio $=$
=>

Q: What is arbitrage profit if the price of Orange is $\$ 125$ instead of $\$ 119.61$ ? How do you create this profit?

|  | \$ in one year <br> Transaction$\quad \underline{\text { Weak }} \quad$ today |  |  |
| :--- | :--- | :--- | :---: |

$\overline{\text { Total }} \quad \overline{\mathbf{0 . 0 0}} \quad \overline{\mathbf{0 . 0 0}}$

Q: What is the arbitrage profit if the price of Orange is $\$ 110$ instead of $\$ 119.61$ ?

| Transaction | $\underline{\text { Stoday }}$ | Win one year <br> Weak | $\underline{\text { Strong }}$ |
| :--- | :--- | :--- | :--- |

Q: What are the possible returns, expected return, and risk premium on Orange if it is correctly priced at $\$ 119.61$ ?

Return on Orange if strong economy $=32.9 \%=$
Return on Orange if weak economy $=-20.6 \%=$
Note:

Q: How should the risk premium on Orange compare to the market (5\%)?

$$
=>
$$

Expected cash flow for Orange $=127=$
Expected return on Orange $=.062=6.2 \%=$
Risk premium on Orange $=.042=$

Ex. Assume that all of the information in the Orange example still holds (Market index trades for $\$ 100$ today and pays, $\$ 75$ or $\$ 139$ a year from today. Risk-free rate equals $2 \%$ ).
Assume also that we can invest in Pineapple which pays $\$ 65$ when the economy is weak and $\$ 129$ when the economy is strong?

Q1: What is the no-arbitrage price for Pineapple?
Q2: What is the arbitrage profit if Pineapple's price is $\$ 95$ or $\$ 80$ ?
Q3: If Pineapple is correctly priced, what are the possible returns, expected return, and risk premium on the stock?

Note:
Equivalent portfolio:

Transaction Weak Strong
Buy Pineapple
Equivalent Portfolio:

Total
Cost of equivalent portfolio $=$
A1: no-arbitrage price of Pineapple $=\$ 90.20$
A2 (\$95): Arbitrage profit if the price of Pineapple is $\$ 95$ instead of the no-arbitrage price of $\$ 90.20$.

|  | \$ in one year <br> Transaction$\quad \underline{\text { Weak today }} \quad$ Strong $\quad$ Transaction |  |  |
| :--- | :--- | :--- | :---: |

$\overline{\text { Total }} \quad \overline{\mathbf{0 . 0 0}} \quad \mathbf{0 . 0 0}$

A2 (\$80): Arbitrage profit if the price of Pineapple is $\$ 80$ instead of the no-arbitrage price of $\$ 90.20$.
$\$$ in one year
Transaction S today Weak Strong Transaction
$\overline{\text { Total }} \overline{+} \quad \overline{\mathbf{0 . 0 0}} \quad \mathbf{0 . 0 0}$

A3: Possible returns, expected return, and risk premium on Pineapple if it is correctly priced at $\$ 90.20$

Return on Pineapple if strong economy $=43 \%=$
Return on Pineapple if weak economy $=-27.9 \%=$
Note: return on Pineapple is more volatile than the market ( $+39 \%$ or $-25 \%$ )
Expected return on Pineapple $=.0755=7.55 \%$
Risk premium on Pineapple $=.0555=$
Note: Risk premium on Pineapple larger than 5\% on market.
E. Transaction cost: cost to trade securities

Note: transaction costs include:

1. commission to broker
2. bid-ask spread: difference between bid price and ask price

Key: Transaction costs lead to the following modifications of earlier definitions:
Normal market $=>$ no arbitrage after transaction costs covered
Law of one price $=>$ difference in prices for equivalent securities must be less than transaction costs
No arbitrage price $=>$ differences between price and the $\mathrm{PV}(\mathrm{CF})$ must be less than transaction costs
Portfolio prices => Difference between the price of a portfolio and the sum of the prices of assets in the portfolio must be less than the transaction costs to build or break apart the portfolio

