Chapter 3: Arbitrage and Financial Decision Making

Fundamental question: What are assets worth?

=> starting point: any two assets that always pay the same cash flows should have the same price

=> if not the case, whoever notices it can make a lot of money very quickly.
=> all mispriced assets will disappear almost immediately as bought (if price too low) or sold (if price too high)

I. Financial Decision Making

A. Steps

1. Identify costs and benefits
   Note: work with accountants, managers, economists, lawyers, etc. to determine costs and benefits
2. Convert costs and benefits to equivalent dollars today
3. Proceed if the value of the benefits exceed the value of the costs

B. Equivalent Dollars (Value) Today

1. When competitive markets exist

a. Definitions and example

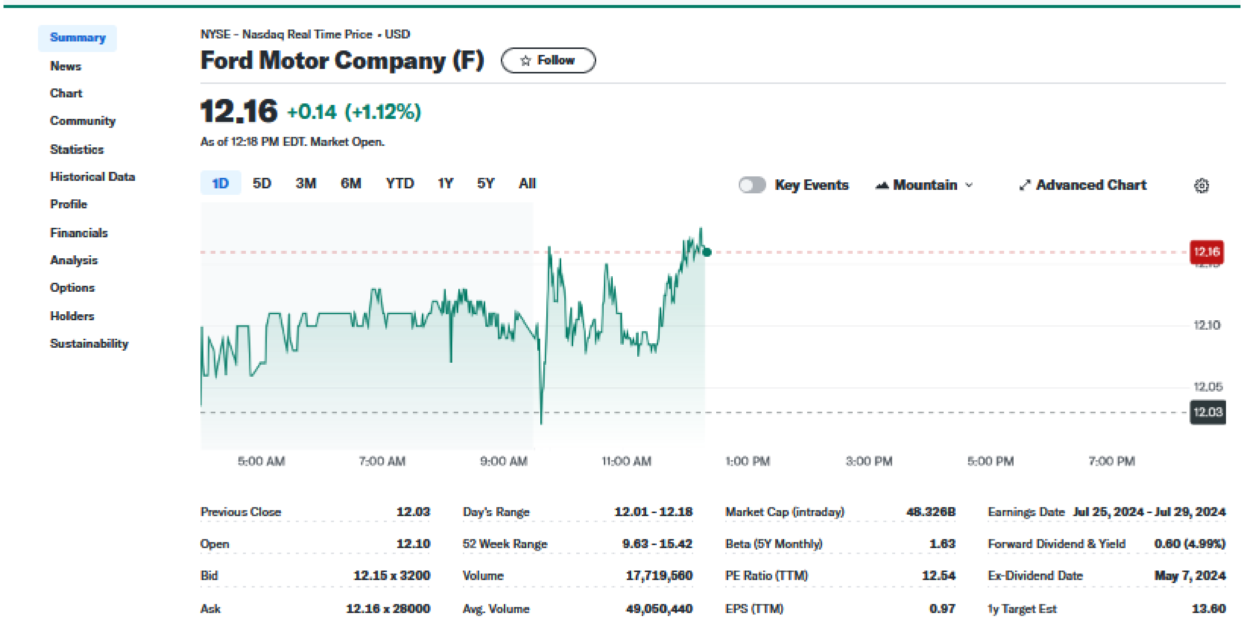
Competitive market: **goods can be bought and sold at the same price**

Q: Do such markets exist?

=> The NYSE is pretty close

Bid price: **highest price at which anyone is willing to buy**
Ask price: **lowest price at which anyone is willing to sell**

=> Ford Example:



Current Ford quote: https://finance.yahoo.com/quote/F

Notes:

1) anyone can submit their own bid or ask price
   => called a limit order

2) anyone submitting a market order takes whatever price exists in the market now

   => if buying, they'll pay whatever the ask price is (the lowest price that anyone is willing to sell for)
   => if selling, they'll get whatever the bid price is (the highest price that anyone is willing to pay).

b. Equivalent value today if competitive market: **market price**

Note: **value doesn't depend on individual preferences or expectations**

Ex. Assume your uncle gives your 100 shares of Ford. What is the gift worth?
Ex. Would you trade your shares for $1000?

Ex. Would you trade your shares for 100 shares of Honda?

Current Honda Quote: https://finance.yahoo.com/quote/HMC

2. When a competitive market does not exist

Note: This is when finance gets more interesting

Ex. Ford dealership on Hwy 84 in Waco (Bird Kultgen)

a. Equivalent value today: **present value of future cash flows**

Notes:

1) **cash flows at different points in time are in different units**
   => **can't combine or compare them**
2) interest rate: **exchange rate across time**
   => **allows us to convert dollar at one point in time to another point in time**
3) interpretation: **present value = amount would need to invest today at the current interest rate to end up with the same cash flow in the future**

C. Making Decisions

1) Accept all positive NPV projects or the highest NPV project if must chose
2) NPV = present value of all cash flows (inflows and outflows)
3) Interpretation of NPV: **wealth created by project**
4) Another way to think about it: **NPV equals the difference between the cost of the project and how much it would cost to recreate a project's cash flows at the current interest rate**
5) Decision doesn't depend on preference for cash today vs. cash in the future

Ex. Assume you have an opportunity to buy land for $110,000 that you will be able to sell for $120,000 a year from today. Further assume that all cash flows are known for sure and that you can borrow or lend at a risk-free interest rate of 4%.

a. Should you buy the land if you have $110,000?

$$\mathbf{NPV = -110{,}000 + \frac{120{,}000}{1.04} = -110{,}000 + 115{,}384.60 = 5384.6}$$

Q: How much would you have to invest at 4% to end up with $120,000 a year from today?

=> $115,384.60

Q: How much better off are you if you buy the land? **$5384.60**

=> $5384.60 equals the **difference between the $100,000 cost to buy the land and the 115,384.60 it would take to recreate the $120,000 a year from today through the financial markets.**

b. Should you buy the land if you have no money?

=> yes

Q: How?

=> **borrow $115,384.60 and buy the land for $110,000**

=> **keep $5384.60 today**
=> **in one year sell the land and use the proceeds to pay off the loan**

Q: Is it realistic to assume that a firm or individual could borrow more than a project costs and to keep the difference?

A: Not really. Rules implemented after the global financial crisis of 2007 – 2008 prevent borrowing more than the value of an asset. Before the financial crisis, some people borrowed more than the value of a house, but most of that involved fraud. But the idea is theoretically sound.

II. Arbitrage and the Law of One Price

A. Introduction and Definitions

1. Arbitrage: **trading to take advantage of price differences between equivalent assets possibly trading in different markets**

Notes:

1) key transactions: **buy low-priced asset and simultaneously sell the equivalent high-priced-asset**
2) equivalent assets: **assets with exactly the same cash flows in all periods under all conditions**
3) **arbitrage requires no investment and creates a riskless payoff today**

Ex. Assume that the price for GE stock is $160 on the New York Stock Exchange and $150 on the NASDAQ.

Arbitrage: **simultaneously buy a share on the NASDAQ and sell a share on the NYSE.**

Arbitrage profit: **$10 today with no risk and no investment**

Q: How many shares want to simultaneously buy and sell?

Ex. Assume the following prices for HMC stock are available on the CBOE and the New York Stock exchange:

Note: HMC stock traded on CBOE and NYSE are equivalent since same exact asset

CBOE				NYSE			
Bid		Ask		Bid		Ask	
Price	Size	Price	Size	Price	Size	Price	Size
25.73	7000	25.76	6000	25.88	26,000	25.89	35,000

Q: What transactions create arbitrage? What is the profit?

Arbitrage: **simultaneously buy shares on the CBOE and sell shares on the NYSE for risk-free profit of $720.**

Note: Arbitrage profit = **6000 x (25.88 – 25.76)**
Q: Why do we use $25.88 and $25.76?
Q: Why not trade more than 6000 shares?
Q: How long will these conditions last?
=> exploiting arbitrage eliminates arbitrage opportunities

Current Quotes:

CBOE: https://www.cboe.com/us/equities/market_statistics/book/hmc/
NYSE (on Yahoo): https://finance.yahoo.com/quote/HMC

2. Normal market: **no arbitrage possible**

Reason should be "normal": **arbitrage will only exist until someone notices it…and a lot of people are looking for such opportunities.**

3. Equivalent assets: **assets with exactly the same cash flows**

4. Law of one price: **equivalent assets trading at the same time in different normal markets must have the same price**

5. Short sales:

1) today: **borrow a security (usually from a broker) and sell it**
2) later: **buy same security and give it back to whoever you borrowed it from**

Notes:

1) **if the security has matured, might pay the cash value rather than buying the security and giving it back**
2) **must make up any cash flows the lender would have received while the security was borrowed**
3) **short seller can buy and return the security at any time**
4) **lender can demand the return of the loaned security at any time**

Ex. Assume you want to short-sell 100 shares of GE today for the market price of $160 per share

1) **borrow 100 shares from your broker and sell them on the NYSE**

Q: Where stand?
=> **owe your broker 100 shares of GE**
=> **have $16,000 (160 x 100) in your brokerage account**

2) assume price falls from $160 to $155

3) Q: How close out short position?
=> **buy 100 shares at $155 per share and give the shares to your broker**

4) assume that while you were short GE paid a dividend of $0.28 per share
=> **must give $28 to your broker.**

5) Profit **=** $972 = **+16,000 – 28 – 15,000**

B. No Arbitrage Prices for Securities

Key: **For there to be no arbitrage, the price of any security must equal the present value of its cash flows**

Ex. Assume you can borrow or deposit in a bank at the risk-free rate of 7% and that a risk-free bond pays $1000 a year from today

$$PV = \frac{\mathbf{1000}}{\mathbf{1.07}} = \mathbf{934.58}$$

Goal in arbitrage: positive cash flow today, no possible net cash flow after today

Basic questions to ask when setting up an arbitrage:

1) What transaction (or set of transactions) is equivalent to the security?
2) Do you want to buy or sell the security?
3) What cash flows does this create?
4) What transaction today offsets the security's cash flows in the future?

Q: What are equivalent transactions?

Transaction	Equivalent Transaction	$ today	$ in one year
Buy bond	**Deposit $934.58**	**-$934.58**	**+$1000**
Short-sell bond	**Borrow $934.58**	**+934.58**	**-$1000**

a) Assume price of bond is $920 (rather than its present value)
=> *arbitrage is possible*

Q: Buy or sell the bond if the price is $920 rather than $934.58?
Buy (buy low and sell high).
Q: What cash flows does this create?
Today = -$934.58; One year= +$1000
Q: What transaction today offsets the security's cash flows in the future?
Borrow $934.58

Table solution:

Transaction(t=0)	$ today	$ in one year	Transaction(t=1)
Buy bond	**-920.00**	**+1000.00**	**Payoff from bond**
Borrow $934.58	**+934.58**	**-1000.00**	**Pay off loan**
Total	**+14.58**	**0.00**	

Arbitrage profit = **$14.58**

b) Assume price of bond is \$950 (rather than its present value)

Q: Buy or sell the bond if the price is \$950 rather than \$934.58?
Sell (buy low and sell high).
Q: What cash flows does this create?
Today = +\$934.58; One year= -\$1000
Q: What transaction today offsets the security's cash flows in the future?
Deposit \$934.58

Table solution:

Transaction (t=0)	\$ today	\$ in one year	Transaction (t=1)
Short-sell bond	**+950.00**	**-1000.00**	**Buy back bond, give to lender**
Deposit \$934.58	**-934.58**	**+1000.00**	**Withdraw from bank**
Total	**+15.42**	**0.00**	

Arbitrage profit = **\$15.42**

=> only way there is no arbitrage: **price = \$934.58**

Notes:

1) investors rushing to take advantage of any arbitrage opportunity will quickly drive the price to \$934.58

2) interest rates are usually extracted from security prices rather than the other way around

Ex. What is usually known: CF_1 = \$1000, Price = \$934.58

$$=> 934.58 = \frac{1000}{1+r} => r = .07 = 7\%$$

3) In a normal market, buying and selling securities has zero NPV

Keys:

a) NPV(buying security) = **PV(CF) - price**

=> in normal market, price = PV(CF)

b) NPV(selling security) = **price – PV(CF)**

=> in normal market, price = PV(CF)

c) otherwise arbitrage possible

C. No Arbitrage Prices of Portfolios

Portfolio: collection of securities

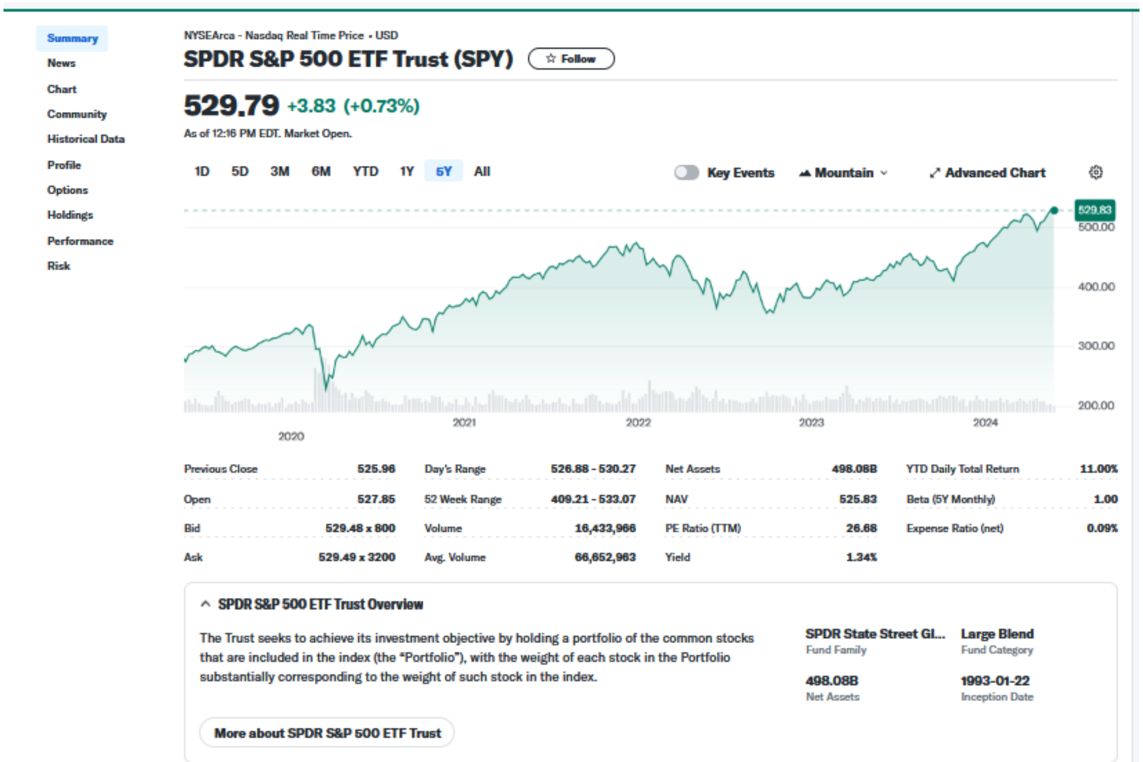
Key: **In a normal market, equivalent portfolios (exactly same cash flows) must have same price**

=> otherwise arbitrage is possible

1. ETF: exchange traded fund

=> **essentially a portfolio of securities that you can trade on an exchange**

Ex. SPDR S&P500 ETF Trust



Current Quotes:

SPY: https://finance.yahoo.com/quote/SPY

S&P500: https://finance.yahoo.com/quote/%5EGSPC

2. Value Additivity: the price of a portfolio must equal the combined values of the securities in the portfolio

=> Price(A+B) = Price(A) + Price(B) (3.5)

Ex. Assume the following:

ETF1 has one share of security A and one share of security B.
ETF2 has one share of security C and one share of security D.
Security A pays $100 a year from today and has a market price of $95.24.
Security B pays $150 a year from today and has a market price of $142.86.
Security C pays $200 a year from today and Security D pays $50 a year from today.

Q: What portfolio is equivalent to ETF1?

Transaction	$ in one year
Buy ETF1	+250.00

Equivalent portfolio:

Buy A	+100.00
Buy B	+150.00
Total	+250.00

=> **buying a share of A and a share of B is equivalent to buying the ETF**

Q: What is the no-arbitrage price be for ETF1?

=> **238.10 = 95.24 + 142.86**

Reason: **ETF1 must have the same price as a portfolio of A and B**

Key to arbitrage with equivalent portfolios with different prices: **buy low and sell high**

Assume price of ETF1 is $220 instead of $238.10

Arbitrage: Buy ETF1, short-sell equivalent portfolio

Transaction (t=0)	$ today	$ in one year	Transaction (t=1)
Buy ETF1	**-220.00**	**+250.00**	**Payoff on ETF**
Short-sell A	**+95.24**	**-100.00**	**Buy back A, return to lender**
Short-sell B	**+142.86**	**-150.00**	**Buy back B, return to lender**
Total	**+18.10**	**0.00**	

Assume price of ETF1 is $245

Arbitrage: short-sell ETF1, buy equivalent portfolio

Transaction (t=0)	$ today	$ in one year	Transaction (t=1)
Short-sell ETF1	**+245.00**	**-250.00**	**Buy back ETF, return to lender**
Buy A	**-95.24**	**+100.00**	**Payoff on A**
Buy B	**-142.86**	**+150.00**	**Payoff on B**
Total	**+6.90**	**0.00**	

=> only way no arbitrage: price of ETF1 = 238.10
=> arbitrage will quickly drive the price of ETF1 to $238.10

Q: What does the market price for ETF2 have to be?

Note: payoff on ETF2 next year: 200 + 50 = 250

Q: What portfolio is equivalent to ETF2?

=> **ETF1**

=> **must be worth 238.10**

Reason: **otherwise arbitrage is possible between ETF1 and ETF2**

2. Value Additivity and Firm Value

Key issues:

=> value of firm = sum of value of individual assets
=> change in value of firm from decision = NPV of decision

III. Appendix to Chapter 3: The Price of Risk

A. Risky Verses Risk-Free Cash Flows

1. Key ideas

1) **investors prefer less risk other things equal**

Reason: for most people a $1 loss is a bigger deal than a $1 gain

2) Risk premium: extra return demanded by investors for holding risky assets instead of Treasuries

=> compensates investors for taking any risk

2. Risk premium on the market

=> **additional return can expect for taking the market's risk**

Note: the market risk premium will increase if:

- the risk of the market increases or,
- if investors become more risk averse

3. Risk premium on a security

Key => Depends on two things:

1) **risk premium on market index**
2) **degree to which security's return varies with market index.**

=> **more varies with market, higher the risk premium**

Ex. Assume the following:

– risk-free interest rate = 2%
– a strong or weak economy is equally likely
– price of the market index: $100
– payoff on stock market index depends on the economy as follows:
weak economy = $75
strong economy = $139

– payoff on Orange Inc. depends on the economy as follows:
weak economy = $95
strong economy = $159

Q: What are the expected cash flow next year, the possible returns, the expected return, and the risk premium on the market?

=> expected cash flow for the market index = 107= $\frac{1}{2}(75) + \frac{1}{2}(139)$
=> return on the market depends on the economy as follows:
Strong: 39% = $\frac{139-100}{100}$
Weak: 0 – 25% = $\frac{75-100}{100}$
=> expected return on the market index: 7% = $\frac{107-100}{100} = \frac{1}{2}(39\%) + \frac{1}{2}(-25\%)$
=> risk premium on the market index = 5% $= 7\% - 2\%$

Q: What is the no-arbitrage price of Orange Inc.?

Q: How does the payoff on Orange compare to the payoff on the market?

=> **Orange always pays $20 more than the market**

Q: How create a portfolio that is equivalent to Orange?

Transaction	$ in one year Weak	Strong
Buy Orange	**+95.00**	**+159.00**
Equivalent Portfolio:		
Buy market index	**+75.00**	**+139.00**
Buy risk-free bond	**+20.00**	**+20.00**
Total	**+95.00**	**+159.00**

Cost to build portfolio that is equivalent to Orange:

=> Cost of equivalent portfolio $= \mathbf{119.61} = 100 + \frac{20}{1.02} = 100 + 19.61$

=> **the price of Orange must equal 119.61 => otherwise arbitrage**

Q: What is arbitrage profit if the price of Orange is $125 instead of $119.61? How do you create this profit?

Transaction	$ today	$ in one year Weak	Strong	Transaction
Short-sell Orange	**+125.00**	**– 95.00**	**– 159.00**	**Buy back Orange, return**
Buy market index	**– 100.00**	**+75.00**	**+139.00**	**Payoff on market**
Buy risk-free bond	**– 19.61**	**+20.00**	**+20.00**	**Payoff on bond**
Total	**+5.39**	**0.00**	**0.00**	

Q: What is the arbitrage profit if the price of Orange is $110 instead of $119.61?

Transaction	$ today	$ in one year Weak	Strong	Transaction
Buy Orange	**– 110.00**	**+95.00**	**+159.00**	**Payoff Orange**
Short-sell market index	**+100.00**	**– 75.00**	**– 139.00**	**Buy back market**
Short-sell risk-free bond	**+19.61**	**– 20.00**	**– 20.00**	**Buy back bond**
Total	**+9.61**	**0.00**	**0.00**	

Q: What are the possible returns, expected return, and risk premium on Orange if it is correctly priced at $119.61?

Return on Orange if strong economy = 32.9% = $\mathbf{\frac{159 - 119.61}{119.61}}$

Return on Orange if weak economy = -20.6% = $\mathbf{\frac{95 - 119.61}{119.61}}$

Note: **return on Orange less volatile than the market (+39% or -25%)**

Q: How should the risk premium on Orange compare to the market (5%)?

=> **should be less**

Expected cash flow for Orange = 127= $\mathbf{\frac{1}{2}(159) + \frac{1}{2}(95)}$

Expected return on Orange= .062 = 6.2% = $\mathbf{\frac{127-119.61}{119.61}} = \mathbf{\frac{1}{2}}(\mathbf{32.9}\%) + \mathbf{\frac{1}{2}}(\mathbf{-20.6}\%)$

Risk premium on Orange = .042 **= .062 – .02**

Ex. Assume that all of the information in the Orange example still holds (Market index trades for \$100 today and pays, \$75 or \$139 a year from today. Risk-free rate equals 2%). Assume also that we can invest in Pineapple which pays \$65 when the economy is weak and \$129 when the economy is strong?

Q1: What is the no-arbitrage price for Pineapple?
Q2: What is the arbitrage profit if Pineapple's price is \$95 or \$80?
Q3: If Pineapple is correctly priced, what are the possible returns, expected return, and risk premium on the stock?

Note: **Pineapple always pays \$10 less than the market.**

Equivalent portfolio:

	\$ in one year	
Transaction	Weak	Strong
Buy Pineapple	**+95.00**	**+159.00**
Equivalent Portfolio**:**		
Buy market index	**+75.00**	**+139.00**
Short-sell risk-free bond	**– 10.00**	**– 10.00**
Total	**+65.00**	**+129.00**

Cost of equivalent portfolio = $\mathbf{90.20 = 100 - \frac{10}{1.02} = 100 - 9.80}$

A1: no-arbitrage price of Pineapple = \$90.20

A2 (\$95): Arbitrage profit if the price of Pineapple is \$95 instead of the no-arbitrage price of \$90.20.

		\$ in one year		
Transaction	\$ today	Weak	Strong	Transaction
Short-sell Pineapple	**+95.00**	**– 65.00**	**– 129.00**	**Buy to cover Pineapple**
Buy market index	**– 100.00**	**+75.00**	**+139.00**	**Payoff on market**
Short-sell risk-free bond	**+9.80**	**– 10.00**	**– 10.00**	**Buy to cover bond**
Total	**+4.80**	**0.00**	**0.00**	

A2 (\$80): Arbitrage profit if the price of Pineapple is \$80 instead of the no-arbitrage price of \$90.20.

		\$ in one year		
Transaction (t = 0)	\$ today	Weak	Strong	Transaction (t = 1)
Buy Pineapple	**– 80.00**	**+65.00**	**+129.00**	**Payoff Orange**
Short-sell market index	**+100.00**	**– 75.00**	**– 139.00**	**Buy to cover market**
Buy risk-free bond	**– 9.80**	**+10.00**	**+10.00**	**Payoff on bond**
Total	**+10.20**	**0.00**	**0.00**	

A3: Possible returns, expected return, and risk premium on Pineapple if it is correctly priced at \$90.20

Return on Pineapple if strong economy = 43% $=\frac{129-90.20}{90.20}$

Return on Pineapple if weak economy = -27.9% $= \frac{65-90.20}{90.20}$

Note: return on Pineapple is more volatile than the market (+39% or -25%)

Expected return on Pineapple = .0755 = 7.55%$= \frac{1}{2}(\mathbf{43}\%) + \frac{1}{2}(\mathbf{-27.9}\%)$

Risk premium on Pineapple = .0555 = **.0755 – .02**

Note: Risk premium on Pineapple larger than 5% on market.

E. Transaction cost: cost to trade securities

Note: transaction costs include:
1. commission to broker
2. bid-ask spread: difference between bid price and ask price

Key: Transaction costs lead to the following modifications of earlier definitions:

Normal market => no arbitrage after transaction costs covered
Law of one price => difference in prices for equivalent securities must be less than transaction costs
No arbitrage price => differences between price and the PV(CF) must be less than transaction costs
Portfolio prices => Difference between the price of a portfolio and the sum of the prices of assets in the portfolio must be less than the transaction costs to build or break apart the portfolio