Micro Seminar 5315

Consumer theory midterm, 2013

Instructions: Answer each question thoroughly. All answers must be legible. Please indicate your solution by circling the answer.

1. (**20 points**) The consumer seeks to maximize the following utility function

$$U(x,y) = -y^{-1} - x^{-1} \tag{1}$$

subject to the budget constraint $I = P_x X + P_y Y$.

   (a) Calculate Marshallian demand for $x(P_x, P_y, I)$. Rigorously show the effect of an increase in the price of x on demand for x.

   (b) It is common to hear someone say that money cannot buy *love* or *happiness*,[1] but what about utility? Rigorously show the effect of an increase in exogenous income on indirect utility using the utility function provided above.

2. (**20 points**) Suppose that a consumer has the following expenditure function:

$$E(P_x, P_y, V) = P_y V + 2 P_x^{\frac{1}{2}} P_y^c \tag{2}$$

   (a) What is the value of the parameter, c? Explain your answer.

   (b) Rewrite the expenditure function with the value of c found in part a. Use Shepherd's Lemma to derive the compensated demand for $y^c(P_x, P_y, V)$.[2] Rigorously show the effect of a change in the price of x on demand. Are these goods net complements, net substitutes, gross complements or gross substitutes?

3. (**15 points**) A consumer has Cobb-Douglas utility, $U = f(x,y) = (xy)^{\frac{1}{2}}$. Each good can only be purchased in markets at prices, P_x and P_y, using exogenous (non-labor) income, I. Assume that (i) $x, y > 0$; (ii) $P_x = 1, P_y = 4, I = 8$.[3] The demand functions are provided for you: $x(P_x, P_y, I) = \frac{I}{2P_x}$ and $y(P_x, P_y, I) = \frac{I}{2P_y}$. The government wishes to collect tax revenue and is choosing between a "per-unit tax" (excise tax) or a lump-sum income tax.

   (a) Calculate the demand for x and y if the government taxes each unit of x at 1.

   (b) Assume government taxes non-labor income, I, by an amount equalling the tax revenue collected under the excise tax from the previous question. Calculate demand for x and y under this lump-sum income tax.

   (c) Which tax regime does the consumer prefer? Why?

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[1] The Beatles wrote, "I don't care too much for money / For money can't buy me love".
[2] You are not required to prove Shepherd's Lemma.
[3] All prices and income are in US dollars for simplicity. There are no externalities or leisure in this problem, either.

4. (**15 points**) Which of the following, if any, are *not* examples of uncompensated demand? Why/why not?

$$y(P_x, P_y, I) = \frac{I}{2P_y P_x} \tag{3}$$

$$y(P_x, P_y, I) = \frac{I}{(P_x P_y)^{\frac{1}{2}} + P_y} \tag{4}$$

$$y(P_x, P_y, I) = \frac{I + P_y - P_x}{2P_x} \tag{5}$$

5. (**10 points**) TRUE or FALSE **and** EXPLAIN: Label each statement TRUE or FALSE, and rigorously justify your answer. You will not receive credit unless you support the answer with a rigorous answer, in other words.

   (a) In a model with two goods, x and y, if x is an inferior good, then x and y are gross complements in response to an increase in the price of y (True/False).

   (b) According to Hicks, most goods in the economy are substitutes (True/False).

6. (**20 points**) Consider the two-good utility maximization model and assume x_1 is a Giffen good. That is, $\frac{\partial x_1}{\partial p_1} > 0$. Prove that $\frac{\partial x_2}{\partial p_1}$ and $\frac{\partial x_2^c}{\partial p_1}$ must be of opposite signs.

BONUS QUESTIONS: The following two questions are bonus questions requiring that you provide a proof of the envelope theorem as part of your answer. Partial credit will be provided for successfully proving the envelope theorem, but no credit will be given if you do not formally prove the envelope theorem.

- (**10 points**): Provide a proof of Shepherd's Lemma and use it to show that one of the properties of expenditure functions is that expenditure functions are concave in prices

- (**10 points**): In question 1, part b, you were asked to calculate the effect of income, I, on indirect utility, $V(P_x, P_y, I)$ for a specific utility function (equation 1). Perhaps, therefore, your answer to 1b was due to the functional form in equation 1, as opposed to a property of indirect utility functions. Examine the question again by calculating $\frac{\partial V(P_x, P_y, I)}{\partial I}$ using the envelope theorem. Your answer should rigorously prove the result as shown in class. Interpret your result while answering the question from 1b (i.e., "does income increase utility?"). Under what conditions will exogenous increases in non-labor income have no effect on consumer utility?