Banking Panics and Policy Responses

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Abstract

We study how banking panics unfold in a version of the Diamond and Dybvig (1983) model with limited commitment. As is well known, the banking authority could eliminate the possibility of a run on the banking system by committing to suspend payments to depositors if a run were to start. Once a run is under way, however, the banking authority will choose to reschedule, rather than suspend, payments. We construct equilibria in which depositors run on the banking system with positive probability, and we show that an equilibrium bank run in this setting is necessarily partial, with only some depositors participating. We also show that a run naturally occurs in waves, with each wave of withdrawals prompting a further policy response from the banking authority. The number of waves that occur in equilibrium is stochastic and can be arbitrarily large.

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1 Introduction

Banking crises often feature a *run* by depositors, that is, an event in which many depositors rush to withdraw their funds from the banking system in a short period of time. Such runs occurred regularly in the United States in the late 19th and early 20th centuries and have occurred more recently in Argentina (in 2001), Russia (in 2004), and elsewhere. Other types of financial institutions and some financial markets have also experienced runs in which investors rush to withdraw funds or sell assets. Competing explanations have been offered for these events. Some observers claim that runs are invariably caused by fundamental factors such as a deterioration of banks' asset positions or an unusually high level of liquidity demand. Others, however, believe that these runs often have a self-fulfilling nature: each individual withdraws because the withdrawals of others threaten the solvency of the banks or the market value of some assets. In this view, a run represents a coordination failure.

A growing literature asks whether this latter explanation is plausible from the standpoint of modern economic theory. Can self-fulfilling runs be explained as equilibrium outcomes of a formal economic model? The answer to this question has important policy implications, particularly regarding the design – and even the desirability – of deposit insurance systems and other elements of the financial safety net. The existing literature has focused predominantly on the potential for self-fulfilling bank runs and has produced mixed results, as evidenced by Green and Lin (2003) and Peck and Shell (2003). In general, however, it is clear that constructing a model in which self-fulfilling runs occur as an equilibrium phenomenon has proven to be difficult.

We depart from this literature by removing the (implicit) assumption that the banking authority can commit to follow a particular course of action in the event of a crisis. Studying an environment without commitment seems natural when considering bank runs and other crises; it amounts to assuming that policy makers are unable to commit not to intervene if an (ex post) improvement in resource allocation is possible. We show that self-fulfilling runs easily emerge in such a setting. Moreover, the run equilibria in our model have a rich structure with several novel features.

To see why the issue of commitment is so important, consider the canonical model of Diamond and Dybvig (1983). Individual agents are unsure about when they will need to consume and, therefore, pool their resources in a bank for insurance purposes. Assume there is no uncertainty about the aggregate "fundamental" demand for withdrawals. In an environment with commitment, the banking authority sets a payment schedule – a complete specification of how much it will give to each depositor who withdraws early – before depositors make their withdrawal decisions. By threatening to suspend payments if too many depositors withdraw early, the banking authority can guarantee the solvency of the banking system.[1] When solvency is guaranteed, it is a dominant strategy for each depositor to wait to withdraw unless she truly needs to consume early. Hence, commitment to an appropriate suspension plan can rule out the possibility of a bank run and can uniquely implement the efficient risk-sharing arrangement.

In an environment without commitment, however, the response of the banking authority to a crisis will be very different. Once the number of early withdrawals exceeds fundamental withdrawal demand, the banking authority realizes that a run is underway. In earlier work (Ennis and Keister, 2009a), we showed that the full suspension policy described above, which calls for completely suspending payments at this point, is not *ex post* efficient. The banking authority knows that if a run is occurring, some of the depositors who have not yet been served have a true need to consume early. Suspending payments means denying consumption to these individuals. A better response is to reschedule payments (often called a *partial suspension*) by offering a smaller – but still positive – payment on further early withdrawals. If depositors anticipate that the banking authority will not completely suspend payments in response to a run, then they recognize that a run may compromise the solvency of the banks. In this way, an *ex post* efficient response to a run may generate *ex ante* incentives for depositors to run.[2]

In this paper, we study an environment in which the banking authority and all depositors fully anticipate and optimally react to each others' behavior, but cannot commit to future actions. We show that when depositors are sufficiently risk averse, there exists an equilibrium of the model in which depositors run on the banking system with positive probability. Despite the simplicity of the environment, the structure of the equilibrium we construct is surprisingly rich. The initial run is necessarily partial, with only some depositors participating. Once the banking authority infers that a run is underway, it will decrease the payment offered on early withdrawals. The run may halt at this point or it may continue, leading the banking authority to announce another, more severe rescheduling of payments. A bank run thus occurs in "waves," with each wave of

[1] In a related model, de Nicolò (1996) shows how run equilibria can be ruled out under commitment without suspending payments by using a priority-of-claims provision on final date resources. Suspension policies have been studied in similar settings by Gorton (1985), Chari and Jagannathan (1988), and Engineer (1989).

[2] Ennis and Keister (2009a) also provides a discussion of institutional features that often shape a government's response to a run, with a focus on events in Argentina in 2001-2 and other recent banking crises.

withdrawals prompting a further reaction by the banking authority. The number of waves that occur in equilibrium is stochastic and can be arbitrarily large.

This dynamic "wave" structure is fundamentally different from the type of bank run studied in the existing literature, where depositors run either *en masse* or not at all. This difference stems, in large part, from a difference in the underlying reason why the banking authority is unwilling to suspend payments. Much of the existing literature studies environments where the total demand for early consumption is random.[3] In such settings, a large number of withdrawals could reflect a high realization of fundamental withdrawal demand rather than a run. Peck and Shell (2003) showed that run equilibria can exist under this approach, but these equilibria have the property that policy makers (or an outside observer) can never distinguish a run from a high realization of fundamental withdrawal demand, even after the fact. To us, it seems implausible to think that throughout a run on the banking system, the authorities remain unsure whether a run is underway or they are simply observing an unusually high level of fundamental withdrawal demand. Bank runs are extreme events that, once fully underway, are easily recognized.

The run equilibria in our model are fully consistent with this view. In equilibrium, the banking authority is able to infer with certainty that a partial run has taken place. The reason it does not fully suspend payments is that doing so would cause substantial hardship for some agents. In addition, the structure of the equilibrium is such that at each decision point, the banking authority is optimistic that the run has ended. This optimism leads it to offer a relatively high degree of risk sharing to the remaining depositors, which, in turn, makes it possible for the run to continue. In this way, our model suggests that the combination of a lack of commitment with optimism on the part of policy makers during a crisis may lie at the root of the problem of self-fulfilling runs. We believe this is a new and potentially important insight into the fundamental causes of financial instability.

Our analysis also contributes to a small but growing literature on discretionary policy and multiple equilibria. Most of the work on time-inconsistency issues has studied situations where the inability of a policy maker to commit leads to an inefficient outcome in the (unique) equilibrium. In our setting, the efficient outcome is always *an* equilibrium. A policy maker with commitment power can rule out other (bank run) equilibria, but a lack of commitment power allows such equi-

[3] See, for example, section IV of Diamond and Dybvig (1983), Postlewaite and Vives (1987), Wallace (1988, 1990), de Nicolò (1996), Green and Lin (2003), Peck and Shell (2003), and Andolfatto et al. (2007).

libria to arise. Hence, our results are more in line with the flood control example in Kydland and Prescott (1977). In that example, a commitment to not invest in flood control would convince private agents to not build on a flood plain. However, if the policy maker cannot commit, there is an equilibrium in which agents build on the flood plain and, as a result, the policy maker ends up investing in flood control.[4] This second type of inefficiency resulting from a lack of commitment power has been studied in the context of fiscal policy by Glomm and Ravikumar (1995) and in the context of monetary policy by Albanesi, et al. (2003) and King and Wolman (2004). Our analysis shows how these same forces naturally generate self-fulfilling bank runs in the well-known Diamond-Dybvig framework.

The rest of the paper is organized as follows. In the next section, we describe the environment and the decisions of depositors for a given payment schedule. In Section 3, we define equilibrium for both the commitment and the no-commitment case. We also show that there exists an equilibrium in which no run occurs and the first-best allocation obtains in each case. In Section 4, we show that bank runs cannot occur in the environment with commitment. Section 5 contains the main result: bank runs *can* occur in the no-commitment case; we also derive some properties of the run equilibria. Sections 6 and 7 contain a discussion of the results and some concluding remarks.

2 The Model

We work with a fairly standard version of the Diamond-Dybvig model with an explicit sequential service constraint. We begin by describing the physical environment and deriving the first-best allocation in this environment.

2.1 The environment

There are three time periods: $t = 0, 1, 2$. There is a continuum of agents, whom we refer to as *depositors*, indexed by $n \in [0, 1]$. Each depositor has preferences given by

$$u(c_1, c_2; \theta_n) = \frac{(c_1 + \theta_n c_2)^{1-\gamma}}{1-\gamma},$$

where c_t is consumption in period t and θ_n is a binomial random variable with support $\Theta = \{0, 1\}$.

[4] See King (2006) for a more formal analysis of this problem.

As in Diamond and Dybvig (1983), we assume that the coefficient of relative risk aversion γ is greater than 1. If the realized value of θ_n is zero, depositor n is *impatient* and only cares about consumption in period 1. A depositor's type θ_n is revealed to her in period 1 and remains private information. Let π denote the probability with which each individual depositor will be impatient. By a law of large numbers, π is also the fraction of depositors in the population who will be impatient.[5] Note that π is non-stochastic; there is no aggregate (*intrinsic*) uncertainty in this model.

The economy is endowed with one unit of the good per capita in period 0. As in Diamond and Dybvig (1983), there is a single, constant-returns-to-scale technology for transforming this endowment into consumption in the later periods. A unit of the good invested in period 0 yields $R > 1$ units in period 2, but only one unit in period 1.

There is also a banking technology that allows depositors to pool resources and insure against individual liquidity risk. The banking technology is operated in a central location. As in Wallace (1988, 1990), depositors are isolated from each other in periods 1 and 2 and no trade can occur among them. However, each depositor has the ability to visit the central location once, either in period 1 or in period 2 and, hence, a payment can be made to her from the pooled resources after her type has been realized. We refer to the act of visiting the central location as *withdrawing* from the banking technology.

Depositors' types are revealed in a fixed order determined by the index n; depositor n discovers her type before depositor n' if and only if $n < n'$. A depositor knows her own index n and, therefore, knows her position in this ordering.[6] Upon discovering her type, each depositor must decide whether or not to visit the central location in period 1. If she does, she must consume immediately; the consumption opportunity in period 1 is short-lived. This implies that the payment a depositor receives from the banking technology cannot depend on any information other than the number of depositors who have withdrawn prior to her arrival. In particular, it cannot depend on the *total* number of depositors who will withdraw in period 1, since this information is not available when individual consumption must take place. This sequential-service constraint follows Wallace

[5] There are well-known technical issues associated with the formal statement of the law of large numbers in an economy with a continuum of agents. We ignore the technical details here and refer the reader to Al-Najjar (2004) for a discussion, references, and a possible way to deal with such issues.

[6] This construction follows Green and Lin (2000) and is a simplified version of that in Green and Lin (2003). None of our results depend on the assumption that depositors know this ordering. Exactly the same results would obtain if depositors made their withdrawal decisions before this ordering is realized (as in Diamond and Dybvig 1983, Peck and Shell 2003, and others), only the details would be more complex in some cases. We further discuss the role of this assumption in Section 6.4.

(1988, 1990) and captures an essential feature of banking: the banking system pays depositors as they arrive to withdraw and cannot condition current payments to depositors on future information.

Under sequential service, the payments made from the banking technology in period 1 can be summarized by a (measurable) function $x : [0,1] \rightarrow \mathbb{R}_{+}$, where the number $x(\mu)$ has the interpretation of the payment given to the μ^{th} depositor to withdraw in period 1. Note that the arrival point μ of a depositor depends not only on her index n but also on the actions of depositors with lower indexes. In particular, μ will be strictly less than n if some of these depositors choose not to withdraw in period 1. In period 2, we can, without loss of generality, set the payment to each depositor equal to an even share of the matured assets in the banking technology.[7] Therefore, the operation of the banking technology is completely described by the function x, which we call the *banking policy*. Feasibility of the banking policy requires that total payments in period 1 not exceed the short-run value of assets, even if all depositors choose to withdraw in that period, that is,

$$\int_0^1 x(\mu)\, d\mu \leq 1. \tag{1}$$

We summarize the behavior of depositor n by a function $y_n : \Theta \rightarrow \{0,1\}$ that assigns a particular action to each possible realization of her type. Here $y_n = 0$ represents withdrawing in period 1 and $y_n = 1$ represents waiting until period 2. We refer to the function y_n as the *withdrawal strategy* of depositor n, and we use y to denote the profile of withdrawal strategies for all depositors.

An *allocation* in this environment consists of an assignment of consumption levels to each depositor in each period. An individual depositor's consumption is completely determined by the banking policy x, the profile of withdrawal strategies y, and the realization of her own type θ_n. We can, therefore, define the (indirect) expected utility of depositor n as a function of x and y, that is,

$$v_n(x,y) = E\left[u\left(c_{1,n}, c_{2,n}; \theta_n\right)\right],$$

where E represents the expectation over θ_n. Different depositors may have different equilibrium utility levels even if they follow the same strategy and have the same realized type because they

[7] In principle, some type of payment schedule could be applied in period 2 as well. However, since depositors are risk averse and all information about their actions has been revealed at this point, it will always be optimal to divide the assets evenly among the remaining patient depositors. Importantly, the type of priority-of-claims provision studied in de Nicolò (1996) would never be used in our setting because it is *ex post* inefficient.

would arrive to withdraw at different points in the period-1 ordering. Define U to be the integral of all depositors' expected utilities, *i.e.*,

$$U(x,y) = \int_0^1 v_n(x,y)\, dn. \tag{2}$$

This expression can be given the following interpretation. Suppose that, at the beginning of period 0, depositors are assigned their index n randomly, with each depositor having an equal chance of occupying each space in the unit interval. Then U measures the expected utility of each depositor before places are assigned. We use U as our measure of aggregate welfare throughout the paper (as in Green and Lin 2000, 2003).

2.2 The first-best allocation

Consider the problem of a benevolent social planner who can observe depositors' types as they become known and can directly control the banking technology and the time of withdrawal by depositors. The planner can choose how much and in which period each depositor consumes, contingent on types and subject to the sequential service restriction described above. We call the allocation this planner would generate the (full information) *first best*.

The problem of finding this allocation can be simplified using the following observations. First, note that the planner would give consumption to all impatient depositors in period 1 and to all patient depositors in period 2. Next, because depositors are risk averse and there is no aggregate uncertainty, depositors of a given type will all receive the same amount of consumption. The problem of finding the first-best allocation thus reduces to choosing numbers c_1 and c_2 to solve

$$\max_{\{c_1,c_2\}} \pi\frac{(c_1)^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma} \tag{3}$$

subject to

$$(1-\pi)c_2 = R\,(1-\pi c_1)$$

and non-negativity constraints. The solution to this simplified problem is

$$c_1^* = \frac{1}{\pi + (1-\pi)\,A} \quad \text{and} \quad c_2^* = \frac{RA}{\pi + (1-\pi)\,A}, \tag{4}$$

where

$$A \equiv R^{\frac{1-\gamma}{\gamma}} < 1. \tag{5}$$

Notice that $RA = R^{\frac{1}{\gamma}} > 1,$ which implies c_2^* is larger than c_1^*; patient depositors consume more than impatient ones. Additionally, $c_1^* > 1$ holds and, hence, this allocation provides liquidity insurance to depositors as described by Diamond and Dybvig (1983). Equivalently, one could have the planner choose a payment schedule x and a profile of withdrawal strategies y to solve

$$\max_{\{x,y\}} U(x,y). \tag{6}$$

subject to the feasibility constraint (1). The solution to (6) sets $y_n(\theta_n) = \theta_n$ for all n and $x(\mu) = c_1^*$ for $\mu \in [0,\pi]$, where c_1^* is as defined in (4).[8]

The first-best allocation described here is the same allocation the planner would choose in an environment without the sequential service constraint, where the planner could first observe all depositor's types and then assign a consumption allocation. In our setting, where there is no aggregate uncertainty, the sequential service constraint is non-binding in the planner's problem. However, as we discuss below, the constraint is an important restriction in the decentralized economy where types are private information.

2.3 The depositors' game

In the decentralized economy, each depositor chooses her withdrawal strategy as part of a non-cooperative game. It will often be useful to fix the banking policy x and look at the game played by depositors under that particular policy. Let y_{-n} denote the profile of withdrawal strategies for all depositors except n. An equilibrium of this game is then defined as follows.

Definition 1: Given a policy x, an *equilibrium of the depositors' game* is a profile of strategies $\widehat{y}(x)$ such that

$$v_n(x, (\widehat{y}_{-n}, \widehat{y}_n)) \geq v_n(x, (\widehat{y}_{-n}, y_n)) \quad \text{for all } y_n, \text{ for all } n.$$

Because they are isolated, depositors do not directly observe each others' actions. Therefore, even though these actions take place sequentially, we can think of depositors as choosing their strategies simultaneously (as in Green and Lin 2003).

The depositors' game has been the focus of the literature on bank runs since Diamond and

[8] Since only the π impatient depositors will withdraw in period 1, the payments for $\mu > \pi$ will not occur and need not be specified. Also, any allocation that differs from the one given here only in the consumption of a set of depositors of measure zero will yield the same value of U and, hence, also be first best. To simplify the presentation, we ignore issues involving sets of measure zero and refer simply to *the* first-best allocation.

Dybvig (1983). For some policies $x,$ this game may not have a unique equilibrium.[9] We use $\widehat{Y}(x)$ to denote the *set* of equilibria associated with the policy $x.$ We say that a *bank run* occurs in an equilibrium $\widehat{y}$ if more than π depositors withdraw in period $1.$ Since all impatient depositors will choose to withdraw in period $1,$ a run occurs if and only if some patient depositors withdraw early, *i.e.*, $\widehat{y}_n(1) = 0$ for a positive measure of depositors.

Diamond and Dybvig (1983) showed how a banking policy resembling a simple demand-deposit contract can implement the (full information) first-best allocation as an equilibrium of this game, even though depositors' types are private information. Suppose the policy is given by

$$x(\mu) = \left\{ \begin{array}{ll} c_1^* & \text{for } \mu \in [0, \widehat{\mu}] \\ 0 & \text{otherwise} \end{array} \right\} \text{ with } \widehat{\mu} = (c_1^*)^{-1}. \tag{7}$$

The value of $\widehat{\mu}$ is the point at which the funds in the banking technology would be completely exhausted in period 1; this policy satisfies the feasibility constraint (1) by construction. Under this policy, each depositor has the option of withdrawing her deposit at face value (c_1^*) in period $1,$ as long as funds are available.

It is fairly easy to see that (i) the strategy profile $y_n(\theta_n) = \theta_n$ is an equilibrium of the depositors' game under this policy and (ii) this equilibrium implements the first-best allocation. In fact, this same result will hold under *any* policy that offers c_1^* to the first π depositors to withdraw. The payments $x(\mu)$ for $\mu > \pi$ are not made under this strategy profile (or under unilateral deviations from it) and are, therefore, irrelevant for the existence of this equilibrium. The converse of this statement is also true: in order for a policy to implement the first-best allocation in the depositors' game, it must be the case that the payment c_1^* is offered to the first π depositors to withdraw. We thus have a sharp characterization of the set of policies capable of implementing the first-best allocation.

Proposition 1 *The policy x implements the first-best allocation as an equilibrium of the depositors' game if and only if it satisfies*

$$x(\mu) = c_1^* \text{ for } \mu \in [0, \pi]. \tag{8}$$

[9] The global games approach of Carlsson and van Damme (1993) has been applied in a variety of settings to generate a unique equilibrium in this type of coordination game. As is clear from Goldstein and Pauzner (2005), however, applying this approach to the Diamond-Dybvig environment requires making strong (and somewhat implausible) assumptions about the investment technology and placing *ad hoc* restrictions on the banking policy.

3 Equilibrium

We now turn our attention to the overall banking game, which includes the determination of the policy x. We assume the banking technology is operated by a benevolent banking authority (BA), whose objective is to maximize the welfare function U. The BA is a reduced-form representation of the entire banking system of the economy, together with any regulatory agencies and other government entities that have authority over the banking system. Our analysis would be exactly the same if there were a group of profit-maximizing banks competing for deposits in period 0 and if the authority to suspend payments in period 1 were held by the (benevolent) government. To keep the presentation simple, and in line with the previous literature, we present the model with this system represented by a single, consolidated entity. We begin our analysis with the total endowment deposited in the banking technology and, hence, under the control of this authority.[10]

3.1 Equilibrium with commitment

We say that the BA has *commitment* if it chooses the entire policy x before depositors make their withdrawal decisions and cannot change any part of the policy later. The previous literature has implicitly assumed commitment. Wallace (1988), for example, views the banking location as a cash machine that is programmed in advance to follow a particular payment schedule. Depositors observe the policy x and, therefore, the depositors' game is a proper subgame of the "overall" banking game. This focus is, naturally, on subgame perfect equilibria, where the BA sets a policy x with the knowledge that the withdrawal strategies will correspond to an equilibrium of the depositors' game generated by x. If there are multiple equilibria of the depositors' game, the BA must have an expectation about which equilibrium will be played; equilibrium of the overall game then requires that this expectation be correct.

As is well known, there cannot be an equilibrium of the overall banking game in which a bank run occurs with certainty. If the BA knew that depositors would run, it would set the policy in such a way that running is not an equilibrium strategy; in other words, it would choose a "run proof" contract (see, for example, Cooper and Ross 1998). A run can only occur in equilibrium

[10] We abstract from what Peck and Shell (2003) call the "pre-deposit game" for simplicity. One can show that if agents were allowed to choose how much of their private endowment to deposit, they would strictly prefer to deposit everything in the banking system as long as the probability of a run is low enough. In this way, our approach is without any loss of generality.

if, at the time it sets its policy, the BA is unsure whether or not a run will occur.[11] To allow for this possibility, we follow the literature in permitting depositors' withdrawal decisions to be conditioned on an extrinsic "sunspot" variable that is not observed by the BA.[12] We assume, without any loss of generality, that the sunspot variable is uniformly distributed on $S = [0,1]$. Each depositor then chooses a strategy $y_n : \Theta \times S \to \{0,1\}$ in which her action is a (measurable) function of the sunspot state. In equilibrium, the BA correctly anticipates the profile of withdrawal strategies y but may not (initially) know the profile of actions because it does not observe the sunspot state s. In particular, the BA may not know whether a run is underway until it has observed enough actions to infer the state.

The BA does know that, in each state, play will correspond to an equilibrium of the depositors' game generated by the chosen policy x. We represent the BA's expectation of depositors' play by a selection $\widehat{y}(x,s)$ from $\widehat{Y}(x)$, that is, a function with $\widehat{y}(x,s) \in \widehat{Y}(x)$ for all x and all s. In other words, the BA expects that if it chooses policy x, depositors will play $\widehat{y}(x,s)$ in state s. An equilibrium of the overall banking game obtains when the BA's policy choice is welfare maximizing given its expectation of depositors' play and, given this choice, the expectation is fulfilled. We formally define an equilibrium of the overall game with commitment as follows.

Definition 2: An *equilibrium with commitment* of the (overall) banking game is a pair (x^*, y^*), together with a selection function $\widehat{y}(x,s) \in \widehat{Y}(x)$ for all x and s, such that

(i) $y^*(s) = \widehat{y}(x^*, s)$ for each s, and

(ii) $\int_0^1 U(x^*, y^*(s))\, ds \geq \int_0^1 U(x, \widehat{y}(x,s))\, ds$ for all x.

This definition can be viewed as a type of correlated equilibrium, using a particular correlating device (which we label 'sunspots') that is asymmetrically observed by depositors and the BA (see Peck and Shell 1991 for this interpretation of correlated equilibrium).

It follows immediately from Proposition 1 that the overall banking game with commitment has an equilibrium in which the first-best allocation obtains in all states. If the BA expects $y_n^*(\theta_n, s) = \theta_n$ to be played, independent of s, by all depositors in response to a policy satisfying (8), then such a policy is clearly an optimal choice for the BA, satisfying condition (ii). Proposition 1 shows that when such a policy is chosen, the strategy profile $y_n(\theta_n, s) = \theta_n$ for all s and n satisfies condition

[11] The issues discussed here are not unique to models of bank runs; they arise in any environment where multiple equilibria are possible and a policymaker makes some decisions before knowing which equilibrium will be played. See Bassetto and Phelan (2008) and Ennis and Keister (2005) for discussions of these issues in models of optimal taxation.

[12] See Diamond and Dybvig (1983), Cooper and Ross (1998), and Peck and Shell (2003).

(i). Hence, we have constructed an equilibrium of the overall banking game in which the first-best allocation obtains in all states.

Corollary 1 *The banking game with commitment has an equilibrium in which the first-best allocation obtains.*

Our question of interest, of course, is whether there exists another equilibrium of the banking game in which some or all patient depositors withdraw in period 1 in some states (i.e., a run equilibrium). The answer to this question depends crucially on the suspension component of the policy, that is, the payments $x(\mu)$ for $\mu > \pi$, and on the BA's ability to commit to the policy. Before addressing the issue of run equilibria, however, we describe the environment without commitment and show that the result in Corollary 1 is unaffected by the absence of commitment.

3.2 Equilibrium without commitment

In an environment without commitment, the banking authority is not able to irrevocably set the payment schedule before depositors choose their withdrawal strategies. Instead, the payment $x(\mu)$ is finally determined only when it is actually made. This approach captures important features of reality. While a banking contract is generally agreed on when funds are deposited, governments routinely reschedule payments during times of crisis. The assumption of the no-commitment case is that the rescheduling plan cannot be fixed in advance; it will be chosen as a best response to whatever situation the banking authority finds itself facing. It is worth emphasizing that the banking authority in our model is completely benevolent; its objective is always to maximize the welfare function U. The assumption in this case, therefore, is simply that the government is unable to commit not to intervene if a crisis is underway and an improvement in resource allocation is possible.

We modify the model presented above to capture the notion of a lack of commitment power in the following way. When choosing a payment $x(\mu)$ for some $\mu > 0$, the BA must clearly recognize that the actions of all previous depositors have already been made. In addition, the BA cannot commit to any payments to later depositors, nor will the choice of $x(\mu)$ affect these future payments.[13] The BA therefore considers the strategies of the remaining depositors to be

[13] With a large number of depositors, the payment to one individual has a negligible effect on total resources and, hence, on subsequent decision problems. Furthermore, the isolation of depositors implies that only the individual receiving the payment $x(\mu)$ directly observes the amount paid; all other depositors must infer the payment using

independent of its choice of $x(\mu)$. In other words, in the environment without commitment, the BA chooses each payment $x(\mu)$ taking the entire strategy profile y^* as given. This is actually a standard formulation of a policy game without commitment; see, for example, the discussion in Cooper (1999, p.137).

The definition of equilibrium for the environment without commitment is therefore as follows.

Definition 3: An *equilibrium without commitment* of the (overall) banking game is a pair (x^*, y^*) such that

$(i)\ y^*(s) \in \widehat{Y}(x^*)$ for all s, and

$(ii)\ \int_0^1 U(x^*, y^*(s))\, ds \geq \int_0^1 U(x, y^*(s))\, ds$ for all x.

Notice the small but important difference between Definitions 2 and 3. In the environment with commitment, the BA recognizes that a change in its policy will lead to a change in the behavior of depositors as specified in the function $\widehat{y}$. Without commitment, in contrast, the BA takes the strategies of depositors as given and must choose a best response to these strategies.

In other words, with commitment the BA can threaten drastic action (such as immediately suspending payments) when faced with a run and depositors know that this threat will be carried out if necessary. Such a response need not be *ex post* optimal; as long as the BA has committed to the action, runs will not occur and the threat will not need to be carried out in equilibrium. Removing the assumption of commitment imposes a form of *credibility* on the BA's threats; a threat to suspend payments will be deemed credible by depositors only if suspending is actually the BA's best response when faced with a run. In other words, our approach involves applying the time consistency notion of Kydland and Prescott (1977) to policies that potentially lie off of the equilibrium path of play.[14]

Before moving on, we note that the reasoning behind Corollary 1 above also applies to the environment without commitment. If the BA expects $y_n(\theta_n, s) = \theta_n$ for all s and n, it will attempt to implement the first-best allocation by choosing a policy satisfying (8). Given such a policy, Proposition 1 shows that this strategy profile is indeed an equilibrium of the depositors' game and,

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the structure of equilibrium. Hence the BA cannot use changes in $x(\mu)$ as a "signal" aimed at influencing the behavior of depositors who have not yet learned their types and whose payments have not yet been determined.

14 The related work of Bassetto (2005) is also concerned with the specification of government policy along potentially off-equilibrium paths and shows how multiplicity of equilibria is more common than previously thought. His approach, however, assumes commitment and only requires that announced policies be *feasible* along all possible paths of play. Condition (1) ensures feasibility in our setup; in particular, suspending payments is always feasible. For us, the ability (or inability) to *commit* to a policy is the critical issue.

hence, we have constructed an equilibrium of the overall banking game.

Corollary 2 *The banking game without commitment has an equilibrium in which the first-best allocation obtains.*

The difference between the environments with and without commitment, therefore, is not related to the ability of the BA to generate the efficient allocation as *an* equilibrium outcome. Rather, the key difference lies in the ability – or inability – of the BA to rule out undesirable allocations as competing equilibrium outcomes. We address this issue formally in the next two sections.

4 The Commitment Case

The central point of Diamond and Dybvig (1983) was that the demand deposit contract described in (7) does not uniquely (or, fully) implement the first-best allocation in the depositors' game. Under this policy, there exists another equilibrium in which all depositors attempt to withdraw in period 1. In this equilibrium, depositors who arrive at the BA before it runs out of funds in period 1 receive c_1^*, while depositors who arrive later (or who deviate and wait until period 2) receive nothing. This equilibrium resembles a run on the banking system and leads to an inefficient allocation of resources.

Could a run occur in an equilibrium of the overall banking game? Diamond and Dybvig (1983) provided a partial answer to this question by showing how a suspension of convertibility clause could render the first-best allocation the *unique* equilibrium outcome of the depositors' game. Suppose that instead of following (7), the BA sets

$$x(\mu) = \left\{ \begin{array}{ll} c_1^* & \text{for } \mu \in [0, \pi] \\ 0 & \text{otherwise} \end{array} \right\}. \tag{9}$$

In other words, suppose the BA announces that after paying c_1^* to a fraction π of depositors in period 1, it will close its doors and refuse to serve any more depositors until period 2. Then a patient depositor will know that, regardless of how many people attempt to withdraw in period 1, the BA will have enough resources to pay her at least c_2^* in period 2. Since $c_2^* > c_1^*$ holds, waiting to withdraw is a strictly dominant strategy for a patient depositor, and the only equilibrium of the depositors' game has $y_n(\theta_n) = \theta_n$ for all n, independent of the sunspot state. This policy thus costlessly eliminates the possibility of a bank run.[15]

The above reasoning implies that an equilibrium of the overall banking game with commitment must lead to the first-best consumption allocation, with impatient depositors receiving c_1^* and patient depositors receiving c_2^* in all states. The BA's equilibrium policy x^* is not uniquely defined, because many policies beside (9) will lead to the same result. However, if the equilibrium allocation had a positive measure of patient depositors withdrawing early in some states of nature, the BA could raise welfare by switching to (9).

Proposition 2 *The first-best allocation obtains in any equilibrium of the banking game with commitment.*

This result shows that under the assumption of commitment, bank runs cannot occur in equilibrium because the BA has a policy tool (suspension of convertibility) that costlessly rules them out.

5 Banking Policy without Commitment

In this section, we investigate the existence of equilibrium bank runs in the environment without commitment. We first show that there cannot be an equilibrium in which all depositors withdraw early in all states or even in only some states. We then derive conditions on parameter values under which there exist "partial run" equilibria, where some patient depositors withdraw early in some states but others always wait. We show that the fraction of depositors withdrawing early in such an equilibrium is stochastic and can be arbitrarily close to one in some states.

5.1 No "full-run" equilibrium

It is fairly easy to see that, even in the environment without commitment, our model cannot have an equilibrium in which all depositors choose to withdraw early with certainty. If the BA expects all depositors to play $y_n = 0$, independent of θ_n and s, its best response is to set $x(\mu) = 1$ for all μ, thereby dividing its assets evenly among the depositors. Under this policy, however, the payment available to a patient depositor who deviates and withdraws in period 2 is $R > 1$, regardless of the number of early withdrawals. Waiting until period 2 is then a dominant strategy for patient

[15] In fact, this result does not require that the BA suspend payments right at π; it is sufficient for the BA to suspend payments at any point where it can still afford to give more than c_1^* to depositors who are paid in period 2. As long as this is true, the actual suspension point chosen does not matter because a suspension never occurs in equilibrium.

depositors and, hence, there cannot be an equilibrium in which these depositors withdraw early.

A slightly more subtle argument shows that there cannot be an equilibrium in which all patient depositors withdraw early in some states but wait until period 2 in the remaining states. To see why, suppose depositors all follow such a strategy, that is,

$$y_n(\theta_n, s) = \left\{ \begin{array}{ll} \theta_n & \text{for } s > s_1 \\ 0 & \text{for } s \leq s_1 \end{array} \right\} \text{ for all } n. \tag{10}$$

for some $s_1 \in (0, 1)$. This type of strategy profile has been discussed extensively in the literature; see, for example, Diamond and Dybvig (1983), Cooper and Ross (1998), and Peck and Shell (2003). Faced with this profile of strategies, the BA's best response would be of the following form. The first π depositors to withdraw provide no information to the BA, since the fraction of depositors withdrawing is at least π in every state. The BA will, therefore, give some common amount c_1 to each of these depositors. The size of the payment c_1 will depend on s_1, of course, but the exact amount is not important for the argument.

The BA recognizes that after π withdrawals have taken place, additional withdrawals in period 1 will only occur in states with $s \leq s_1$, in which case all depositors will withdraw early. Therefore, the BA will set the payments $x(\mu)$ for $\mu > \pi$ so as to evenly divide its remaining assets among the remaining depositors, since this is the best response to a run should one occur. Each of these depositors would then receive

$$x(\mu) = c_{1d} \equiv \frac{1 - \pi c_1}{1 - \pi} \text{ for } \mu > \pi,$$

where the d subscript indicates that this payment results in an even division of the BA's remaining assets. Given this payment schedule, does the strategy profile in (10) represent an equilibrium of the depositor's game? The answer is 'no' because the payment available to a patient depositor who deviates and withdraws in period 2 in states $s \leq s_1$ is Rc_{1d}, which is strictly greater than c_{1d}. A patient depositor with $n > \pi$ would, therefore, prefer to wait until period 2 to withdraw. A patient depositor with $n \leq \pi$ may or may not prefer to wait, depending on the relative sizes of c_1 and Rc_{1d}, but either way the strategy profile (10) is not consistent with equilibrium behavior. We summarize this argument in the following proposition.

Proposition 3 *The strategy profile* (10) *cannot be part of an equilibrium of the banking game without commitment.*

5.2 A partial-run equilibrium

The result in Proposition 3 leaves open the possibility of a *partial run* equilibrium, in which some depositors follow (10) and others do not. Based on the discussion above, it seems promising to look for an equilibrium in which depositors who would arrive relatively late in period 1 choose to wait if they are patient, while depositors who would arrive relatively early choose to withdraw regardless of their type. Specifically, suppose the strategy profile of depositors is given by

$$\begin{aligned} &\text{For } s > s_1: \quad y_n(\theta_n, s) = \theta_n \qquad\qquad \text{for all } n \\ &\text{For } s \leq s_1: \quad y_n(\theta_n, s) = \left\{ \begin{array}{c} 0 \\ \theta_n \end{array} \right\} \quad \text{for} \left\{ \begin{array}{c} n \leq \pi \\ n > \pi \end{array} \right\}. \end{aligned} \tag{11}$$

In this subsection, we derive conditions under which this profile is part of an equilibrium of the overall banking game.

We construct this equilibrium in two steps. First, we derive the BA's best response to the strategy profile in (11); let $\widehat{x}$ denote the best-response policy. We then ask under what conditions the profile in (11) is an equilibrium of the depositors' game generated by $\widehat{x}$. We derive a necessary and sufficient condition for this to be the case, and we show that the condition holds when s_1 is small enough and γ is large enough.

We calculate the BA's best response to (11) by working backward, considering first the payments $x(\mu)$ for $\mu > \pi$. Let ψ denote the per-capita amount of resources the BA has left after the first π withdrawals, that is,

$$\psi = \frac{1 - \int_0^\pi x(\mu)\, d\mu}{1 - \pi}.$$

The BA recognizes that the payments for $\mu > \pi$ will only take place in states $s \leq s_{1.}$ If these payments are made, therefore, the BA knows that (i) a run will have occurred, meaning that the first π withdrawals were made by a mix of patient and impatient depositors, but (ii) all additional withdrawals in period 1 will be made by depositors who are truly impatient. The total fraction of depositors withdrawing in period 1 will, therefore, be $\pi + \pi(1-\pi) = 1 - (1-\pi)^2$.[16]

Because depositors are risk averse, the BA will offer a common payment to all of the (impatient) depositors who withdraw after π. We denote this payment $c_{1,2}$, where the latter subscript indicates that the payment is associated with the 2nd "stage" of the payment schedule. The BA will also give

[16] Note that the withdrawals $\mu > 1 - (1-\pi)^2$ will never be made under the strategy profile in (11) and, hence, the best-response levels for these payments are not determined.

a common payment $c_{2,2}$ to the (patient) depositors who withdraw in period 2. These payments will be chosen to maximize the BA's objective function (2) and hence will solve the following problem

$$\max_{\{c_{1,2},c_{2,2}\}} \pi\frac{(c_{1,2})^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_{2,2})^{1-\gamma}}{1-\gamma} \tag{12}$$

subject to

$$(1-\pi)c_{2,2}=R\left[\psi-\pi c_{1,2}\right]$$

and non-negativity constraints. Notice the similarity between this problem and (3). The strategy profile in (11) implies that when a run occurs, it halts after π withdrawals have been made. From that point onward, only impatient depositors withdraw in period 1 and, therefore, the BA is able to implement the first-best *continuation* allocation, given the per-capita amount ψ of resources remaining.

The solution to this problem is given by

$$\widehat{c}_{1,2}=\psi\frac{1}{\pi+(1-\pi)A} \quad \text{and} \quad \widehat{c}_{2,2}=\psi\frac{RA}{\pi+(1-\pi)A} \tag{13}$$

where A is as defined in (5). Here we see that the first-best continuation allocation after π withdrawals resembles the overall first-best allocation (4), but with the payments scaled by the available resources per capita ψ. Let V denote the value of the objective in (12) evaluated at the solution, that is

$$V(\psi)=\pi\frac{(\widehat{c}_{1,2})^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(\widehat{c}_{2,2})^{1-\gamma}}{1-\gamma},$$

or, using (13) and (5),

$$V(\psi)=(\pi+(1-\pi)A)^{\gamma}\frac{\psi^{1-\gamma}}{1-\gamma}.$$

We next ask how the BA will set the payments to the first π depositors who withdraw. The BA does not know whether these payments will go to only impatient depositors, as will happen if $s>s_1$, or to a mix of patient and impatient depositors participating in a run, as will occur if $s\leq s_1$. Regardless of which case applies, however, the BA will want to give the same payment to all π depositors. Any payment schedule for which $x(\mu)$ is not constant for (almost) all $\mu\leq\pi$ is strictly dominated by another policy that makes the same total payment to these depositors (leaving ψ unchanged), but divides the resources evenly among them.

Therefore, the BA will set $x(\mu)=c_1$ for $\mu\in[0,\pi]$, where c_1 is chosen to solve the following problem.

$$\max_{\{c_1,c_2\}} (1-s_1)\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_2)^{1-\gamma}}{1-\gamma}\right)+s_1\left(\pi\frac{(c_1)^{1-\gamma}}{1-\gamma}+(1-\pi)V(\psi)\right) \tag{14}$$

subject to

$$\begin{aligned}(1-\pi)c_2 &= R(1-\pi c_1) \quad \text{and} \\ \psi &= \frac{1-\pi c_1}{1-\pi}.\end{aligned}$$

The solution to this problem is

$$\widehat{c}_1=\frac{1}{\pi+(1-\pi)A_1} \quad \text{and} \quad \widehat{c}_2=\frac{RA_1}{\pi+(1-\pi)A_1}, \tag{15}$$

where

$$A_1=\left((1-s_1)R^{1-\gamma}+s_1\left[\pi+(1-\pi)A\right]^{\gamma}\right)^{\frac{1}{\gamma}}.$$

It is straightforward to show that $RA_1>1$ holds, so that $\widehat{c}_2$ is larger than $\widehat{c}_1$. In other words, if a run does not occur (that is, if $s>s_1$), then depositors withdrawing in period 2 will receive more than depositors withdrawing in period 1. Also, when $s_1>0$ and, hence, a run is possible, it can be shown that $\widehat{c}_{1,2}$ is smaller than $\widehat{c}_1$; that is, depositors who withdraw in period 1 after it becomes clear that a (partial) run has taken place suffer a "discount" relative to depositors who were earlier in line. Summarizing, the BA's best response to the profile of withdrawal strategies (10) is given by

$$\widehat{x}(\mu)=\left\{\begin{array}{c}\widehat{c}_1\\ \widehat{c}_{1,2}\end{array}\right\} \text{ for } \mu\in\left\{\begin{array}{c}[0,\pi]\\ \left(\pi,1-(1-\pi)^2\right]\end{array}\right\}. \tag{16}$$

We next ask if the strategy profile in (11) is an equilibrium of the depositors' game generated by $\widehat{x}$. In other words, if the BA were to follow the payment scheme in (16), would each depositor find it optimal to follow (11) if she believed others would do so? Impatient depositors will always choose to withdraw in period 1, so we only need to consider the actions of patient depositors. In states $s>s_1$, a patient depositor receives $\widehat{c}_2$ if she waits until period 2 to withdraw, but receives $\widehat{c}_1$ if she deviates and withdraws early. Since $\widehat{c}_2>\widehat{c}_1$ holds, waiting to withdraw is clearly the optimal choice in these states.

In states $s\leq s_1$, the payment a patient depositor receives if she chooses to withdraw early depends on her index n. For a patient depositor with $n>\pi$, the choice is between $\widehat{c}_{1,2}$ if she withdraws early and $\widehat{c}_{2,2}$ if she waits. Since $RA > 1$, it is optimal for her to wait, as specified by (11). What about patient depositors with $n \leq \pi$? Such a depositor will also receive $\widehat{c}_{22}$ if she waits until period 2, but will receive the "pre-rescheduling" payment, $\widehat{c}_1$, if she withdraws early. She will choose to follow (11) and withdraw early if $\widehat{c}_1 > \widehat{c}_{22}$; using (15) and (13), this inequality can be shown to be equivalent to

$$f(\gamma, s_1) \equiv R\left((1 - s_1)\frac{R^{1-\gamma}}{\left(\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}\right)^{\gamma}} + s_1\right) < 1. \tag{17}$$

If this condition holds, the profile of withdrawal strategies (11) represents an equilibrium of the depositors' game generated by the policy $\widehat{x}$. Since $\widehat{x}$ is, by construction, the BA's best response to (11), we have constructed an equilibrium of the (overall) banking game without commitment.

When will (17) hold? If the parameters R, γ, and π are such that

$$f(\gamma, 0) = R\frac{R^{1-\gamma}}{\left(\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}\right)^{\gamma}} < 1, \tag{18}$$

then, by continuity, (17) will hold if s_1 is small enough. In other words, if condition (18) holds, we can use the above construction to generate an equilibrium in which the first π depositors run with positive probability. We have, therefore, proven the first of our results on the existence of a bank run equilibrium.

Proposition 4 *If* (18) *holds, there exists an equilibrium of the banking game without commitment in which a fraction π of depositors run on the banking system with positive probability.*

Notice that for any given values of R and π, condition (18) will hold if γ is large enough. In other words, if depositors are sufficiently risk averse, the partial-run equilibrium described above will exist.[17]

5.3 A run equilibrium with two waves

We showed in Section 5.1 that there cannot be an equilibrium in which all depositors run with certainty because the BA, anticipating the run, will divide its resources in such a way that patient

[17] Gu (2008) studies a model with demand-deposit contracts and generates a partial-bank-run equilibrium by having depositors observe imperfectly correlated sunspot signals. In her setting, a partial run occurs in some states and a full run in others. In our environment, in contrast, only partial runs are observed; a full run cannot occur in any state.

depositors would rather wait to withdraw. Similarly, the partial-run equilibrium described above cannot continue beyond the first π withdrawals with certainty, because the BA would again be able to anticipate the continuing nature of the run and would choose (as part of the ex post optimal policy) a payment schedule that actually convinces patient depositors to wait.

It is, however, possible for the run to continue with positive probability. In this subsection we discuss one such equilibrium. This equilibrium has the property that, after the first π depositors have withdrawn during a run, the run may either halt, as in the previous subsection, or the crisis may "deepen" as a second wave of patient depositors withdraws early after the rescheduling of payments. In the latter case, a fraction π of the remaining depositors will withdraw before the BA is able to infer that the run has not stopped. At this point, the BA will choose to reschedule payments again and (in the equilibrium we construct here) the run will halt. In the following subsection, we state and prove a more general result that includes the equilibrium discussed here as a special case.

Consider the following profile of withdrawal strategies:

$$\begin{array}{lll} \text{for } s \geq s_1: & y_n(\theta_n, s) = \theta_n & \text{for all } n \\ \text{for } s \in [s_2, s_1): & y_n(\theta_n, s) = \left\{ \begin{array}{c} 0 \\ \theta_n \end{array} \right\} & \text{for} \left\{ \begin{array}{c} n \leq \pi \\ n > \pi \end{array} \right\} \\ \text{for } s < s_2 & y_n(\theta_n, s) = \left\{ \begin{array}{c} 0 \\ \theta_n \end{array} \right\} & \text{for} \left\{ \begin{array}{c} n \leq 1-(1-\pi)^2 \\ n > 1-(1-\pi)^2 \end{array} \right\} \end{array} \tag{19}$$

for some $s_1 > s_2 > 0$. In this profile, there are two sets of states associated with a run on the banking system. For values of s in $[s_2, s_1)$, a fraction π of depositors will run, as in the previous subsection. For s below s_2, however, these depositors are joined by a fraction π of the remaining depositors.

What is the BA's best response to the strategy profile in (19)? Without going into the details of the calculations, we can see that it must be of the form

$$\widehat{x}(\mu) = \left\{ \begin{array}{c} c_1 \\ c_{1,2} \\ c_{1,3} \end{array} \right\} \text{ for } \left\{ \begin{array}{l} \mu < \pi \\ \mu \in \left(\pi, 1-(1-\pi)^2\right] \\ \mu > 1-(1-\pi)^2 \end{array} \right\}. \tag{20}$$

The reasoning behind the form of (20) is exactly the same as that behind (16) in the previous section. When the first π withdrawals are taking place, the BA is unsure whether these withdrawals are being made only by impatient depositors or a run is underway. It assigns probability s_1 to the latter case. Regardless of this probability, however, it will choose to offer a common payment c_1 on all of these withdrawals. This payment level can be found by solving a problem similar to (14), but with the value function in the second term of the objective modified to reflect the richer structure of the strategy profile (19) (see the proof of Proposition 5 in the appendix for details).

If more than π withdrawals take place in period 1, the BA will recognize that a run is underway and will reschedule payments. At this point, however, the BA is unsure whether the run will halt, with all additional period 1 withdrawals being made by impatient depositors, or if it will continue. The run will halt if $s \in [s_2, s_1)$ and will continue if $s < s_2$; hence, the BA assigns (conditional) probability s_2/s_1 to the event that the run continues. Based on this probability, the BA will choose to give a common payment $c_{1,2}$ to the next $\pi(1-\pi)$ depositors who withdraw. Similarly, if more than $1-(1-\pi)^2$ withdrawals take place in period 1, the BA will be able to infer that $s < s_2$. In this case it will solve a problem similar to (12) to find the optimal payment $c_{1,3}$.

The remaining question is whether or not the withdrawal strategies (19) are an equilibrium of the depositors' game generated by the policy (20). Would each individual depositor be willing to follow the strategy in (19) if she expected all others to do so? The answer will be affirmative if and only if the payments induced by the policy (20) satisfy

$$c_1 \leq c_2, \quad c_{1,2} \leq c_{2,2,} \text{ and } c_{1,3} \leq c_{2,3,} \tag{21}$$

as well as

$$c_1 \geq c_{2,2}, \quad c_1 \geq c_{2,3}, \quad \text{and } c_{1,2} \geq c_{2,3}. \tag{22}$$

The inequalities in (21) guarantee that if a run is not currently underway when a patient depositor has the opportunity to withdraw (either because a run never started or because it has halted), she will be willing to wait until period 2. The first inequality applies to states $s \geq s_1,$ where each depositor receives c_1 if she withdraws in period 1 and c_2 if she waits until period 2. The second applies to states $s \in [s_2, s_1)$ and depositors $n > \pi$; in this case a run has occurred but has halted and these depositors will receive either $c_{1,2}$ in period 1 or $c_{2,2}$ in period $2.$ Similarly, the third inequality applies to states $s < s_2$ and depositors $n > 1-(1-\pi)^2$. It can be shown, by deriving expressions similar to (13) and (15), that these inequalities always hold.

The inequalities in (22) guarantee that a patient depositor is willing to participate in the run if one is underway when she has the opportunity to withdraw. The first inequality guarantees that depositors with $n \leq \pi$ are willing to run in states $s \in [s_2, s_1)$, while the second ensures that these same depositors are willing to run in states $s < s_2$. The third inequality guarantees that depositors with n between π and $1-(1-\pi)^2$ are willing to run in states $s < s_2$. If all of these inequalities hold, every depositor will choose to follow (19) if she expects all others to do so and, hence, that strategy profile is an equilibrium of the depositors' game.

Whether or not the inequalities in (22) hold will depend on the cutoff states s_1 and s_2, which have a large impact on the payments that the BA chooses. It can be shown that there exist $s_1 > s_2 > 0$ such that all of these inequalities hold if and only if condition (18) holds. In other words, the condition on the parameters R, γ, and π that guarantees the existence of an equilibrium in which a fraction π of depositors choose to run in some states also guarantees that there is an equilibrium in which $1-(1-\pi)^2$ choose to run in some states. Rather than presenting the details of these calculations, we move directly to our main result, which includes the two-wave run equilibrium as a special case.

5.4 Run equilibria with many waves

Nothing in the above discussion requires that a run must end with certainty after a second wave of early withdrawals. In fact, the same type of reasoning can be used to construct an equilibrium in which a run may occur in any finite number of waves and hence, for some states, the run can involve a fraction of depositors that is very close to one (that is, almost all the depositors in the system). The main proposition of the paper states this result.

Proposition 5 *If* (18) *holds, then for any $\lambda < 1$ there exists an equilibrium of the banking game without commitment in which the fraction of depositors withdrawing in period* 1 *is greater than λ with positive probability.*

The proof is presented in Appendix A. Given some $\lambda < 1$, the first step is to determine the number of waves a run would have to go through in order to involve at least a fraction λ of the depositors. An equilibrium with the required number of waves is then constructed following the type of approach used in the previous subsection. After each wave, the BA is unsure if the run will halt or continue and it reschedules payments in a way that reflects this uncertainty. These rescheduled payments are then shown to satisfy the analog of the conditions in (22) and, hence, they permit the run to continue in some states.

An interesting feature of the run equilibrium identified in Proposition 5 is that, even if nearly all depositors end up withdrawing in period 1, the BA remains "optimistic" throughout the period that the run has already ended. As discussed above, if the BA ever believed that a full run was underway, it would reschedule payments in such a way that the remaining depositors would choose not to run. The only way a run can continue (or even get started) is if the BA is fairly optimistic and, therefore, sets the payment on early withdrawals relatively high. This fact implies that bank runs *must* occur in waves in our environment, with the run likely to end after each wave.

Peck and Shell (2003) study a model with aggregate uncertainty about the fraction of impatient depositors and construct examples of equilibria in which *all* depositors run. In these equilibria, the banking authority remains optimistic that it is observing an unusually large realization of the fraction of impatient depositors rather than a run and, hence, believes that the withdrawals will likely stop soon. In this sense, the aggregate uncertainty in their model plays the role of the wave structure of equilibrium in ours.

The two approaches have fundamental differences, however. In their setting, the banking authority can never know for certain whether or not a run has occurred, even after the fact. In the examples they construct, the event in which all depositors are impatient is much more likely than a run. We do not believe it is plausible to characterize events in the U.S. in the early 1930s or in Argentina in 2001 as possibly resulting from a spike in the fundamental demand for liquidity. Once underway, a run on the banking system is easily recognized. Our model has this property: when more that pi withdrawals take place, the BA correctly infers that a run has taken place. Its optimism is not about whether or not a run has *occurred*, but rather about whether or not the run will *continue* after payments are rescheduled.

6 Discussion

6.1 The probability of a run

The results presented above show that there exist equilibria in which runs occur with positive probability. How large can the equilibrium probability of a run be? This question is easiest to answer for a single-wave run. For that case, we can solve (17) as an equality to find the cutoff probability $\overline{s}$ such that a run equilibrium exists for any $s_1 < \overline{s}$. Doing so yields

$$\overline{s} = \frac{\frac{1}{R} - B}{1 - B},$$

where

$$B = \frac{R^{1-\gamma}}{\left(\pi + (1-\pi) R^{\frac{1-\gamma}{\gamma}}\right)^{\gamma}}.$$

The value of B is strictly decreasing in γ and, hence, $\overline{s}$ is strictly increasing in γ. In other words, when depositors are more risk averse, the maximum probability of a run is higher. Figure 1 plots the area where $s_1 < \overline{s}$ is satisfied.

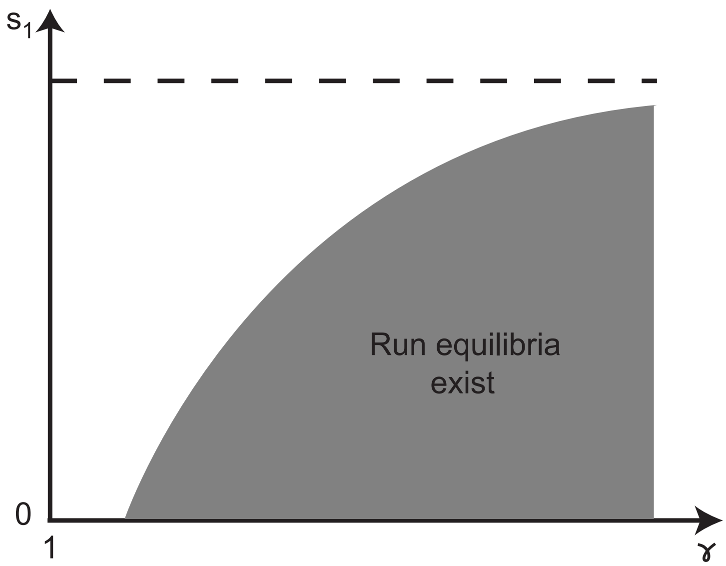

Figure 1: The maximum probability of a run

In the limiting case where γ goes to infinity, $\overline{s}$ converges to R^{-1}. While it is not clear what parameter values should be considered "realistic" in our stylized model, this calculation nevertheless demonstrates that runs need not be rare events in this environment. If, for example, $R = 1.1$, then when depositors are very risk averse there exists an equilibrium in which the probability of a run is greater than 90%.

6.2 The degree of risk aversion

Figure 1 also shows that, for any given values of R and π, the single-wave run equilibrium exists for some values of s_1 when γ is large enough. In other words, bank runs can occur in equilibrium whenever depositors are sufficiently risk averse. In fact, this statement applies to the many-wave run equilibrium constructed in Proposition 5 as well, since both single- and many-wave run equilibria exist if and only if (18) holds. We state this result as a corollary.

Corollary 3 *Given R and π, if depositors are sufficiently risk averse, bank runs can occur with positive probability in an equilibrium without commitment.*

How risk averse must depositors be? Condition (18) holds for some values of R and π if and only if $\gamma > 2$. In other words, the curve in Figure 1 always begins to the right of $\gamma = 2$, and there are values of R and π for which it begins arbitrarily close to $\gamma = 2$. Even this requirement can be weakened, however, through a relatively straightforward modification of the model. Cooper and Ross (1998) studied a two-technology specification of the Diamond-Dybvig model with costly liquidation (see also Ennis and Keister 2006). While they ruled out suspensions of convertibility and payment reschedulings by assumption, they showed how having two technologies and a non-trivial portfolio choice weakens the requirement on risk aversion needed for the run equilibrium to exist. The same is true in our model.

Suppose that, in period 0, the BA had to divide its resources between a liquid investment, which yields a return of 1 in either period 1 or 2, and an illiquid investment that yields R in period 2 but only $1 - \tau$ in period 1, where $\tau \geq 0$ represents a liquidation cost. When $\tau = 0$, the liquid asset will not be used and the model reduces to the one studied here. It can be shown that increasing the liquidation cost shifts the boundary in Figure 1 upward, increasing the set of run probabilities consistent with equilibrium for any given γ. Furthermore, as the liquidation cost becomes large, the starting point of the boundary approaches the origin of the diagram. In other words, for any level of risk aversion $\gamma > 1$, there exists a level of the liquidation cost that permits a run to occur with positive probability.

6.3 The fraction of impatient depositors

An examination of (18) shows that it cannot hold when there are very few impatient depositors. In the extreme case where π is set to zero, the value of $f(\gamma, 0)$ is $R > 1$ and the condition is clearly violated. By continuity, it is also violated for values of π close to zero. Using the fact that f is strictly decreasing in π delivers the following result.

Corollary 4 *Given R and γ, there exists $\overline{\pi} \in (0, 1]$ such that bank runs can occur with positive probability in an equilibrium without commitment if and only if $\pi > \overline{\pi}$.*

Intuitively, when π is small the BA will discover whether or not a run is underway fairly quickly. This allows it to reschedule payments before it has given away many resources during a run. The BA will also discover quickly if the run has halted or if it continues following each wave. Because the BA is able to make inferences frequently and adjust payments accordingly, it will be able to retain a relatively large amount of resources for making payments in period 2. A depositor who sits out the run will then receive a relatively large payment and, as a result, running will not be equilibrium behavior. Viewed this way, Corollary 4 says that the rate at which the BA is able to infer the sunspot state must be sufficiently slow for the possibility of bank runs to arise.

6.4 Depositors' information sets

Following Green and Lin (2000, 2003), we have assumed that depositors know the order in which they would arrive at the bank if they chose to withdraw early. This approach differs from the original Diamond-Dybvig model and most of the subsequent literature, where a depositor first chooses whether or not to withdraw and then is assigned a place in line. The known-ordering approach simplifies our analysis above because, together with the law of large numbers, it implies that each depositor knows precisely how much she will receive if she withdraws early; in other models this payoff is random. Our results, however, do not depend critically on depositors knowing the order. In particular, the type of run equilibria we construct can also be shown to exist in an environment that is closer to the original Diamond-Dybvig model. In this subsection, we briefly describe how this works.

Consider an environment where withdrawal decisions are initially made without any information about the ordering. Depositors who choose to withdraw are randomly assigned places in line. Suppose, however, that when a rescheduling of payments is announced, depositors who are in line but have not yet been served are able to re-evaluate their decision to withdraw. This is the key feature of the alternative environment: a depositor may not know how much she will receive if she withdraws early, but if the amount is less than the "standard" payment then she is able to change her mind. In this way, she can effectively discern the withdrawal payment she would receive before making a final decision.

It is fairly straightforward to construct a run equilibrium with a single wave in this modified environment. The BA follows the policy in (16). All depositors initially attempt to withdraw. After a proportion π of agents withdraw, the banking authority discovers that a run is underway and reschedules payments. In response, the remaining patient depositors in line reevaluate their decision and decide to wait until period 2. This behavior represents an equilibrium under exactly the same conditions as in our analysis above. In particular, if (18) holds, then the initial run can occur with positive probability in equilibrium.

Run equilibria with many waves can be constructed in this alternative environment in a similar fashion. To decide the specifics of the payment schedule, the banking authority needs to form expectations over the possible reaction of depositors. In formalizing this process it is convenient to introduce explicit coordination devices. In particular, suppose that after each partial suspension is announced, the remaining depositors in line can condition their actions on the realization of a new sunspot variable. The run could then halt in some states, but in others it could continue until the next phase of the suspension plan is announced. At that point, yet another sunspot variable is realized and the process can repeat any number of times.

Decision making in this alternative environment has a more dynamic nature, with actions being decided in stages rather than all at once. While this property is perhaps appealing on intuitive grounds, it also raises the usual complications associated with dynamic games, including the need to specify agents' beliefs along off-equilibrium paths of play. In addition, the idea that depositors can change their decisions based on the payments offered by the BA is somewhat at odds with the isolation assumption. One rather appealing aspect of our formulation is that it captures the important features of the "dynamic" story without introducing unnecessary complications.

The fact that our results do not depend on whether or not depositors have information about their position in the withdrawal order stands in stark contrast with the previous literature. Green and Lin (2003) have shown how giving depositors this type of information can, in some circumstances, eliminate the type of run equilibrium studied by Peck and Shell (2003).[18] The discussion here shows that our results are independent of such informational details, which is another appealing aspect of our model.

[18] See also Andolfatto *et al.* (2007) and Ennis and Keister (2009b).

7 Concluding Remarks

The issues of commitment, credibility, and time-inconsistency are pervasive in economics and have been studied extensively. In banking theory, however, the importance of these issues has received relatively little attention, apart from often informal treatments of bank bailouts.[19] In this paper, we analyze the role of commitment in banking policies designed to respond to the possibility of a run on the banking sector. We study a setting in which bank runs would never occur under commitment because, in that case, the threat to suspend payments in response to a run convinces depositors not to run in the first place. In contrast, equilibrium bank runs *can* occur in this same setting when policy makers cannot commit to future actions. These run equilibria take an interesting, and perhaps realistic, form. In particular, the total size of a run in these equilibria is stochastic; after each wave of withdrawals, the run may halt or it may deepen as more depositors withdraw.

A large number of papers have addressed applied questions related to bank runs and financial crises using versions of the Diamond-Dybvig model.[20] In order to obtain a run equilibrium in a tractable way, these papers place ad hoc restrictions on the banking contract, such as not allowing payments to be suspended until banks' assets are totally depleted. This approach has obvious drawbacks, including the fact that the results of such an exercise may depend critically on what restrictions are imposed. The model presented here offers an alternative. There are no restrictions on contracts other than those imposed by the physical environment, and yet the model is highly tractable. Bank run equilibria are readily constructed and have interesting dynamic features. Moreover, studying environments without commitment seems natural when considering bank runs and other types of crises. For these reasons, we believe our model will prove useful for studying a wide range of issues related to banking and financial instability.

Instead of placing ad hoc restrictions on contracts, some papers have studied models with aggregate uncertainty about fundamental withdrawal demand. Suspending payments is less attractive in such settings because the banking authority does not know the proper point at which to suspend. We consider an environment without aggregate uncertainty in order to keep our model analytically tractable, but it is not essential for our results. Changing the model so that the fraction of depositors who are impatient is random will complicate matters, but our insights will remain valid as long as

[19] Two notable exceptions are Mailath and Mester (1994) and Acharya and Yorulmazer (2007), both of which deal with credibility issues in policies regarding bank closure.

[20] See, for example, Temzelides (1997), Cooper and Ross (1998), Allen and Gale (2000) and Chang and Velasco (2001), to name only a few.

the support of the distribution is not too large. What is important for our analysis is that there is an upper bound on the level of normal withdrawal demand, and that suspending payments to depositors once this bound is reached would rule out the possibility of a self-fulfilling bank run. In any such setting, the credibility of the threat to suspend comes into question and the issues highlighted in this paper are relevant.

The willingness of a banking authority to suspend payments in reality is likely to depend on how long the banking system needs to remain closed. In the model, the time lapse between periods 1 and 2 corresponds to the maturity time of banks' investments. More generally, it can be thought of as the time necessary for banks to liquidate their portfolios without incurring significant losses. If this period is fairly short, the fraction of depositors who need "early" access to their funds may be rather small, which would correspond to a low value of π in the model. Our result in Corollary 4 shows that if π is low enough, the banking authority *can* uniquely implement the first-best allocation. In other words, if only a short suspension of payments is required, an inability to commit to a policy is less likely to cause problems. The longer the time period involved, however, the greater is the need for additional early payments and, hence, the more the banking authority will deviate from the commitment solution.

This reasoning suggests that a decrease in the liquidity of the assets held by banks, as has been observed during the recent market turmoil, might increase banks' susceptibility to a run. Similarly, it suggests that a banking system that is perceived to have fundamental weaknesses – in particular, some uncertainty about asset values that will take time to resolve – should be more susceptible to a run than a system that is fundamentally sound. It was clear to observers that the banking crisis in Argentina in 2001, for example, was not likely to be sorted out quickly, which undoubtedly made a strict suspension of payments more difficult and may have contributed *ex ante* to individuals' decisions to run. Formalizing this argument would require a more fully dynamic model and seems a promising avenue for future research.

Studying suspension policies in a longer-horizon setting would introduce other interesting issues. It is well known, for example, that reputational concerns can substitute for commitment in some settings (see Stokey 1991 and Chari and Kehoe 1990). The extent to which the desire to build a reputation for being "tough" in the face of a run would enable the banking authority to credibly suspend payments (and thereby rule out runs) is an interesting question. The answer will likely depend on how, if ever, the reputation is tested given that bank runs potentially lie off the equilibrium path. While these difficult issues are beyond the scope of the present paper, we believe that our analysis provides a critical first step by highlighting their relevance. Once it is recognized that suspension of convertibility policies may not be time consistent even in simple settings, issues of both static and dynamic credibility become important. Our analysis using the classic model of Diamond and Dybvig should serve as a useful benchmark for future work on the issue.

Appendix A. Proof of Proposition 5

Proposition 5: *If* (18) *holds, then for any* $\lambda < 1$ *there exists an equilibrium of the banking game without commitment in which the fraction of depositors withdrawing in period* 1 *is greater than* λ *with positive probability.*

The proof is constructive. Let K be the smallest integer such that

$$1 - (1-\pi)^{K+1} > \lambda$$

holds. Consider the strategy profile

$$\begin{array}{lll} \text{for } s \geq s_1: & y_n(\theta_n, s) = \theta_n & \text{for all } n \\ \text{for } s \in [s_{k+1}, s_k): & y_n(\theta_n, s) = \left\{ \begin{array}{c} 0 \\ \theta_n \end{array} \right\} & \text{for } n \left\{ \begin{array}{c} \leq \\ > \end{array} \right\} 1 - (1-\pi)^k \end{array} \tag{23}$$

for $k = 1, \ldots K$, where

$$1 > s_1 > \ldots > s_K > s_{K+1} \equiv 0.$$

Under this strategy profile, the fraction of depositors withdrawing in period 1 is $1-(1-\pi)^{K+1}$ with probability $s_K > 0$. Therefore, if we can show that (23) is part of an equilibrium of the banking game without commitment, the proposition will be proved. We break this task into two steps, which are addressed in separate lemmas below. First, Lemma 1 derives the BA's best response to this strategy profile, which we denote $\widehat{x}$. Lemma 2 then shows that when (18) holds, we can choose the numbers s_k such that (23) is an equilibrium of the depositors' game generated by $\widehat{x}$. The result in the proposition follows immediately from these two lemmas.

Lemma 1 *The BA's best response to* (23) *is*

$$\widehat{x}(\mu) = \left(\prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi) A_j} \right) \frac{1}{A_k} \ \ \textit{for } \mu \in \left(1 - (1-\pi)^{k-1}, 1 - (1-\pi)^k \right],$$

where

$$A_k = \left((1-q_k) R^{1-\gamma} + q_k (\pi + (1-\pi) A_{k+1})^{\gamma} \right)^{\frac{1}{\gamma}}, \quad \textit{for } k = 1, \ldots, K+1. \tag{24}$$

Proof: We work backwards. Define ψ_K to be the per-capita resources remaining after $1-(1-\pi)^K$

withdrawals have been made, that is,

$$\psi_K = \frac{1 - \int_0^{1-(1-\pi)^K} x(\mu)\,d\mu}{(1-\pi)^K}.$$

We first derive the payments $x(\mu)$ for $\mu \in \left(1-(1-\pi)^K, 1-(1-\pi)^{K+1}\right]$.[21] The BA recognizes that under (23) these payments will only be made in states $s < s_K$ and that all of these payments, in the event they are made, will go to impatient depositors. The remaining patient depositors will wait until period 2 to withdraw. Because depositors are risk averse, the BA will choose to give the same amount to all impatient depositors; we denote this amount $c_{1,K+1}$, where the latter part of the subscript indicates that these payments would apply after there have been K waves of withdrawals and the run has halted. Let $c_{2,K+1}$ denote the payment that the remaining patient depositors will receive in period 2. These payment amounts will be chosen to solve

$$\max_{c_{1,K+1}, c_{2,K+1}} \pi \frac{(c_{1,K+1})^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(c_{2,K+1})^{1-\gamma}}{1-\gamma} \tag{25}$$

subject to

$$(1-\pi)c_{2,K+1} = R\left[\psi_K - \pi c_{1,K+1}\right]$$

and non-negativity constraints. Notice that this problem resembles that for finding the first-best allocation, but with per-capita resources set to ψ_K instead of 1. The solution is

$$\widehat{c}_{1,K+1} = \psi_K \frac{1}{\pi + (1-\pi) A_{K+1}} \quad \text{and} \quad \widehat{c}_{2,K+1} = \psi_K \frac{R A_{K+1}}{\pi + (1-\pi) A_{K+1}}, \tag{26}$$

where

$$A_{K+1} \equiv R^{\frac{1-\gamma}{\gamma}} < 1. \tag{27}$$

Let V_{K+1} denote the value of the objective in (25) evaluated at the solution, that is

$$V_{K+1}(\psi_K) = \pi \frac{(\widehat{c}_{1,K+1})^{1-\gamma}}{1-\gamma} + (1-\pi)\frac{(\widehat{c}_{2,K+1})^{1-\gamma}}{1-\gamma},$$

or, substituting in (26),

$$V_{K+1}(\psi_K) = (\pi + (1-\pi) A_{K+1})^{\gamma} \frac{(\psi_K)^{1-\gamma}}{1-\gamma}.$$

[21] Under (23), there are no circumstances in which the payments associated with $\mu \geq 1-(1-\pi)^{K+1}$ will be made. The best-response levels for these payments are, therefore, indeterminate and do not matter for our analysis.

Next, we consider the payments in the interval

$$\mu \in \left(1-(1-\pi)^{k-1}, 1-(1-\pi)^{k}\right] \quad \text{for any } k \in \{1, \ldots, K\}.$$

These payments will be made in states $s \leq s_{k-1}$. Unlike in the previous case, the BA is not sure if these payments will go only to impatient depositors, as will occur if $s \in [s_k, s_{k-1})$, or to a mix of patient and impatient depositors during a continued run, as will occur if $s < s_k$. Regardless of which case applies, however, the BA will want to give the same payment to all depositors who withdraw in this interval. In other words, any payment schedule for which $x(\mu)$ is not constant for (almost) all μ in this interval is strictly dominated by another policy that makes the same total payments to these depositors, but divides the resources evenly among them. Let $c_{1,k}$ denote the payment given to depositors withdrawing in this interval in period 1. Let $c_{2,k}$ denote the payment that will be received by patient depositors in period 2 if there are no further withdrawals in period 1, that is, if $s \in [s_k, s_{k-1})$.

Before we write the optimization problem for choosing these payment levels, we introduce some notation to simplify the statement of the problem. First, define ψ_{k-1} to be the amount of resources per capita that remain after $1-(1-\pi)^{k-1}$ withdrawals in period 1, that is,

$$\psi_{k-1} = \frac{1-\int_0^{1-(1-\pi)^{k-1}} x(\mu)\, d\mu}{(1-\pi)^{k-1}} \quad \text{for } k = 1, \ldots, K.$$

Straightforward calculations then yield the following relationship between $\psi_{k-1,}$ the payments $c_{1,k}$, and the per-capita resources ψ_k remaining after these payments are made,

$$\psi_k = \frac{\psi_{k-1} - \pi c_{1,k}}{1-\pi}. \tag{28}$$

Next, define

$$q_k = \frac{s_k}{s_{k-1}} = \text{Prob}\left[s < s_k \mid s < s_{k-1}\right] \quad \text{for } k = 1, \ldots, K,$$

with $s_0 \equiv 1$. In other words, q_k is the probability that the run will continue into the k^{th} wave, given that it has lasted for $k-1$ waves. Finally, let $V_k\left(\psi_{k-1}\right)$ denote the average expected utility of depositors with $n > 1-(1-\pi)^{k-1}$ conditional on $s < s_{k-1}$. In other words, V_k measures the expected utility of depositors who have not yet been served when the BA discovers that the run has at least $k-1$ waves. Then the BA will choose the payment $c_{1,k}$ to solve

$$\max_{c_{1,k},c_{2,k}} (1-q_k)\left(\pi\frac{(c_{1,k})^{1-\gamma}}{1-\gamma}+(1-\pi)\frac{(c_{2,k})^{1-\gamma}}{1-\gamma}\right)+q_k\left(\pi\frac{(c_{1,k})^{1-\gamma}}{1-\gamma}+(1-\pi)V_{k+1}(\psi_k)\right)$$

subject to

$$(1-\pi)c_{2,k}=R\left[\psi_{k-1}-\pi c_{1,k}\right],$$

(28), and non-negativity constraints. The first term in the objective function represents utility in the event that the run halts after $k-1$ waves. In this case, the remaining impatient depositors all receive $c_{1,k}$ and the remaining patient depositors receive $c_{2,k}$ in period 2. The second term represents utility in the event that the run continues into the k^{th} wave, which occurs with probability q_k. In this case, the first π depositors to withdraw (a mix of impatient and patient depositors) will receive $c_{1,k}$. The remaining depositors will receive payments after the next phase of the suspension takes effect; the utility of these depositors is captured by the value function V_{k+1}.

Solving this problem recursively backward, substituting the value function for each value of k into the problem for $k-1$ yields

$$\widehat{c}_{1,k}=\psi_{k-1}\tfrac{1}{\pi+(1-\pi)A_k},\quad \widehat{c}_{2,k}=\psi_{k-1}\tfrac{RA_k}{\pi+(1-\pi)A_k},\quad \text{and}$$

$$V_k\left(\psi_{k-1}\right)=\left(\pi+(1-\pi)A_k\right)^{\gamma}\tfrac{\left(\psi_{k-1}\right)^{1-\gamma}}{1-\gamma},$$

where A_k is given in (24). We can then replace the ψ_k terms as follows. Since $\psi_0=1$ (by definition), we have

$$\widehat{c}_{1,1}=\frac{1}{\pi+(1-\pi)A_1}.$$

Then we can calculate the amount of resources remaining after the first π withdrawals

$$\psi_1=\frac{1-\pi\widehat{c}_{1,1}}{1-\pi}=\frac{A_1}{\pi+(1-\pi)A_1},$$

and use this amount to find the optimal payment levels following the first partial suspension

$$\widehat{c}_{1,2}=\frac{A_1}{\pi+(1-\pi)A_1}\frac{1}{\pi+(1-\pi)A_2}\quad\text{and}\quad\widehat{c}_{2,2}=\frac{A_1}{\pi+(1-\pi)A_1}\frac{A_2}{\pi+(1-\pi)A_2}R.$$

Continuing this process forward yields

$$\psi_k=\prod_{j=1}^{k}\frac{A_j}{\pi+(1-\pi)A_j}$$

and

$$\widehat{c}_{1,k} = \left(\prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi) A_j} \right) \frac{1}{A_k} \text{ and } \widehat{c}_{2,k} = \left(\prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi) A_j} \right) R, \tag{29}$$

which establishes the Lemma. ■

Lemma 2 *If* (18) *holds, there exist* $1 > s_1 > \ldots s_K > 0$ *such that* (23) *is an equilibrium of the depositors' game generated by* $\widehat{x}$.

Proof: Since impatient depositors will always choose to withdraw early, we only need to check the optimal behavior of a depositor when she is patient. The strategies in (23) are individually optimal if

$$\left. \begin{array}{lll} (a) & \widehat{c}_{1,j} \geq \widehat{c}_{2,k} & \text{for } j = 1, \ldots k-1 \\ (b) & \widehat{c}_{1,k} \leq \widehat{c}_{2,k} & \end{array} \right\} \quad \text{for } k = 1, \ldots, K+1.$$

The inequalities on line (a) imply that patient depositors are willing to participate in the run. If the run lasts for $k-1$ waves, then a patient depositor who chooses not to run will receive $\widehat{c}_{2,k}$. A patient depositor who withdraws early receives $\widehat{c}_{1,j}$ for some $j < k$ that depends on her index n. If each of these inequalities hold, then all patient depositors who have an opportunity to withdraw during the run will choose to do so. The inequalities on line (b) are often referred to as the *incentive compatibility* constraint. They imply that if a run is not underway, or has halted before a depositor is served, then a patient depositor will be willing to wait and withdraw in period 2.

We examine line (b) first. From (27) we have $RA_{K+1} = R^{\frac{1}{\gamma}} > 1$. Then, using (24), we have

$$RA_k = \left((1-q_k) R + q_k \left(\pi R + (1-\pi) RA_{k+1} \right)^{\gamma} \right)^{\frac{1}{\gamma}}, \quad \text{for } k = 1, \ldots, K.$$

Applied recursively from $k = K$ down to $k = 1$, this expression demonstrates that

$$RA_k > 1 \quad \text{for } k = 1, \ldots, K+1.$$

It then follows immediately from (29) that (b) holds.

Next, we examine line (a). First, from (29) we have

$$\widehat{c}_{1,j+1} = \frac{A_j}{\pi + (1-\pi) A_{j+1}} \widehat{c}_{1,j}. \tag{30}$$

It is straightforward to show that

$$\frac{A_j}{\pi + (1-\pi)\,A_{j+1}} < 1 \quad \text{for } j = 1, \ldots, K.$$

Equation (30) therefore shows that in each wave of the partial suspension, the payment received by depositors is smaller than in the previous wave, an intuitive result. More importantly, this result also implies that instead of checking the $k-1$ inequalities on line (a) for each value of k, we only need to check the last one:

$$\widehat{c}_{1,k-1} \geq \widehat{c}_{2,k} \quad \text{for } k = 2, \ldots, K+1.$$

This inequality can be written as

$$\widehat{c}_{1,k-1} = \left(\prod_{j=1}^{k-1} \frac{A_j}{\pi + (1-\pi)\,A_j} \right) \frac{1}{A_{k-1}} \geq \left(\prod_{j=1}^{k} \frac{A_j}{\pi + (1-\pi)\,A_j} \right) R = \widehat{c}_{2,k},$$

which can be reduced to

$$(A_k R)^{\gamma} \left((1 - q_{k-1}) \frac{R^{1-\gamma}}{(\pi + (1-\pi)\,A_k)^{\gamma}} + q_{k-1} \right) < 1 \quad \text{for } k = 2, \ldots, K+1. \tag{31}$$

By replacing the A_k terms recursively, using (24), we have K inequalities involving only the parameters R, γ, π, and the (endogenous) probabilities $q_1, \ldots, q_K$. The question is under what conditions these probabilities can be chosen so that all K inequalities hold.

Suppose we set $q_k = 0$ for all k. Then $A_k = R^{\frac{1-\gamma}{\gamma}}$ for all k and (31) reduces to the same inequality for all values of k :

$$R^{\frac{1}{\gamma}} \frac{R^{\frac{1-\gamma}{\gamma}}}{\pi + (1-\pi)\,R^{\frac{1-\gamma}{\gamma}}} < 1,$$

which is exactly condition (18). Since the inequalities (31) are clearly continuous in the variables q_k, we therefore know that when (18) holds, there exists a number $q > 0$ such that (31) holds for all k if we set $q_k = q$ for all k. We can then back out the cutoff states $s_1, \ldots, s_K$ by

$$\begin{aligned} s_1 &= q \\ s_k &= q s_{k-1} = q^k \quad \text{for } k = 2, \ldots, K. \end{aligned}$$

Since $s_K > 0$ holds, we have established the lemma. $\blacksquare$

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