## **CHAPTER 3**

### Valuing Bonds

## **Answers to Problem Sets**

1. a. Does not change

b. Price falls

c. Yield rises.

2. a. If the coupon rate is higher than the yield, then investors must be

expecting a decline in the capital value of the bond over its remaining life. Thus, the bond’s price must be greater than its face value.

1. Conversely, if the yield is greater than the coupon, the price will be below

face value and it will rise over the remaining life of the bond.

3. The yield over 6 months is 3.965/2 = 1.79825%.

Therefore, PV = 3/1.0179825 + 3/1.01798252 +…. + 103/1.017982534 = 130.37

4. Yields to maturity are about 4.3% for the 2% coupon, 4.2% for the 4% coupon, and 3.9% for the 8% coupon. The 8% bond had the shortest duration (7.65 years), the 2% bond the longest (9.07 years).

5. a. Fall (e.g., 1-year 10% bond is worth 110/1.1 5 100 if r 5 10% and is worth

110/1.15 = 95.65 if r = 15%).

b. Less (e.g., See 5a).

c. Less (e.g., with r = 5%, 1-year 10% bond is worth 110/1.05 = 104.76).

d. Higher (e.g., if r = 10%, 1-year 10% bond is worth 110/1.1 = 100, while 1-

year 8% bond is worth 108/1.1 = 98.18).

1. No, low-coupon bonds have longer durations (unless there is only one

period to maturity) and are therefore more volatile (e.g., if r falls from 10% to 5%, the value of a 2-year 10% bond rises from 100 to 109.3 (a rise of 9.3%). The value of a 2-year 5% bond rises from 91.3 to 100 (a rise of

9.5%).

6. a. Spot interest rates. Yield to maturity is a complicated average of the

separate spot rates of interest.

1. Bond prices. The bond price is determined by the bond’s cash flows and

the spot rates of interest. Once you know the bond price and the bond’s

cash flows, it is possible to calculate the yield to maturity.

7. a. 4%

b. PV = $1,075.44

8. a. PV 

b. PV 

c. Less (it is between the 1-year and 2-year spot rates).

9. a. r1 = 100/99.423 – 1 = .58%; r2 = (100/97.546).5 – 1 = 1.25%; r3 =

(100/94.510).33 -.1 = 1.90%; r4 = (100/90.524).25 – 1 = 2.52%.

b. Upward-sloping.

c. Lower (the yield on the bond is a complicated average of the separate spot rates).

10. a. Price today is 108.425; price after 1 year is 106.930.

b. Return = (106.930 1 8)/108.425 - 1 = .06, or 6%.

1. If a bond’s yield to maturity is unchanged, the return to the bondholder is

equal to the yield.

11. a. False. Duration depends on the coupon as well as the maturity.

b. False. Given the yield to maturity, volatility is proportional to duration.

c. True. A lower coupon rate means longer duration and therefore higher volatility.

d. False. A higher interest rate reduces the relative present value of (distant) principal repayments.

12.

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13. 7.01% (the extra return that you earn for investing for two years rather than one is 1.062/1.05 – 1 = .0701).

1. a. Real rate = 1.10/1.05 – 1 = .0476, or 4.76%

b. The real rate does not change. The nominal rate increases to 1.0476 x 1.07 – 1 = .1209, or 12.9%.

15. With annual coupon payments:

€92.64

16. a. 

b.

|  |  |  |  |
| --- | --- | --- | --- |
| Interest  rate | PV of  Interest | PV of  Face value | PV of Bond |
| 1.0% | $5,221.54 | $9,050.63 | $14,272.17 |
| 2.0% | 4,962.53 | 8,195.44 | 13,157.97 |
| 3.0% | 4,721.38 | 7,424.70 | 12,146.08 |
| 4.0% | 4,496.64 | 6,729.71 | 11,226.36 |
| 5.0% | 4,287.02 | 6,102.71 | 10,389.73 |
| 6.0% | 4,091.31 | 5,536.76 | 9,628.06 |
| 7.0% | 3,908.41 | 5,025.66 | 8,934.07 |
| 8.0% | 3,737.34 | 4,563.87 | 8,301.21 |
| 9.0% | 3,577.18 | 4,146.43 | 7,723.61 |
| 10.0% | 3,427.11 | 3,768.89 | 7,196.00 |
| 11.0% | 3,286.36 | 3,427.29 | 6,713.64 |
| 12.0% | 3,154.23 | 3,118.05 | 6,272.28 |
| 13.0% | 3,030.09 | 2,837.97 | 5,868.06 |
| 14.0% | 2,913.35 | 2,584.19 | 5,497.54 |
| 15.0% | 2,803.49 | 2,354.13 | 5,157.62 |

17. Purchase price for a 6-year government bond with 5 percent annual coupon:



Price one year later (yield = 3%):



Rate of return = [$50 + ($1,091.59 – $1,108.34)]/$1,108.34 = 3.00%

Price one year later (yield = 2%):



Rate of return = [$50 + ($1,141.40 – $1,108.34)]/$1,108.34 = 7.49%

18. The key here is to find a combination of these two bonds (i.e., a portfolio of bonds) that has a cash flow only at t = 6. Then, knowing the price of the portfolio and the cash flow at t = 6, we can calculate the 6-year spot rate.

We begin by specifying the cash flows of each bond and using these and their yields to calculate their current prices:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Investment | Yield | C1 | . . . | C5 | C6 | Price |
| 6% bond | 12% | 60 | . . . | 60 | 1,060 | $753.32 |
| 10% bond | 8% | 100 | . . . | 100 | 1,100 | $1,092.46 |

From the cash flows in years one through five, we can see that buying two 6% bonds produces the same annual payments as buying 1.2 of the 10% bonds. To see the value of a cash flow only in year six, consider the portfolio of two 6% bonds minus 1.2 10% bonds. This portfolio costs:

($753.32 × 2) – (1.2 × $1,092.46) = $195.68

The cash flow for this portfolio is equal to zero for years one through five and, for year 6, is equal to:

(1,060 × 2) – (1.2 × 1,100) = $800

Thus:

$195.68 × (1 + r6)6 = 800

r6 = 0.2645 = 26.45%

19. Downward sloping. This is because high coupon bonds provide a greater proportion of their cash flows in the early years. In essence, a high coupon bond is a ‘shorter’ bond than a low coupon bond of the same maturity.

20. a.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Year | Discount Factor | Forward Rate | | |
| 1 | 1/1.05 = 0.952 |  |  |  |
| 2 | 1/(1.054)2 = 0.900 | (1.0542 /1.05) – 1 = 0.0580 = 5.80% | | |
| 3 | 1/(1.057)3 = 0.847 | (1.0573 /1.0542 ) – 1 = 0.0630 = 6.30% | | |
| 4 | 1/(1.059)4 = 0.795 | (1.0594 /1.0573 ) – 1 = 0.0650 = 6.50% | | |
| 5 | 1/(1.060)5 = 0.747 | (1.0605 /1.0594 ) – 1 = 0.0640 = 6.40% | | |

b. i. 5%, two-year note:



1. 5%, five-year bond:



1. 10%, five-year bond:



c. First, we calculate the yield for each of the two bonds. For the 5% bond, this means solving for r in the following equation:



r = 0.05964 = 5.964%

For the 10% bond:



r = 0.05937 = 5.937%

The yield depends upon both the coupon payment and the spot rate at the time of the coupon payment. The 10% bond has a slightly greater proportion of its total payments coming earlier, when interest rates are low, than does the 5% bond. Thus, the yield of the 10% bond is slightly lower.

d. The yield to maturity on a five-year zero coupon bond is the five-year spot rate, here 6.00%.

e. First, we find the price of the five-year annuity, assuming that the annual payment is $1:



Now we find the yield to maturity for this annuity:



r = 0.05745 = 5.745%

1. The yield on the five-year note lies between the yield on a five-year zero-coupon bond and the yield on a 5-year annuity because the cash flows of the Treasury bond lie between the cash flows of these other two financial instruments during a period of rising interest rates. That is, the annuity has fixed, equal payments, the zero-coupon bond has one payment at the end, and the bond’s payments are a combination of these.
2. Assuming we are calculating the durations as of February 2009, the strip’s

duration equals

, and the modified duration equals 6/1.02 = 5.88. At a semi-annual yield of 1.5%, the price of the 6-year strips equals 91.424, and at a semi-annual yield of 2.5%, the price of the strips equals 86.151. The difference in the prices, 5.273 is 5.94% of the price of the strips. This is close to the 5.88 duration, and the difference is due to the first-order approximation of the price change provided by duration.

To calculate the duration for the 4% bonds, consider the following table similar to Table 3.3:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 |
| Cash Payment | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 102 |
| PV at 2% ytm | 1.980 | 1.961 | 1.941 | 1.922 | 1.903 | 1.884 | 1.865 | 1.847 | 1.829 | 1.811 | 1.793 | 90.520 |
| fraction value | 0.018 | 0.018 | 0.017 | 0.017 | 0.017 | 0.017 | 0.017 | 0.017 | 0.016 | 0.016 | 0.016 | 0.814 |
| year x fraction of value | 0.009 | 0.018 | 0.026 | 0.035 | 0.043 | 0.051 | 0.059 | 0.066 | 0.074 | 0.081 | 0.089 | 4.882 |

Total PV = 111.26, and the duration = 5.432. The modified duration equals 5.432/1.02 = 5.325.

The price of the 4% coupon bond at 1.5% and 2.5% equals 114.294 and 108.310, respectively. This price difference, 5.984, is 5.38% of the original price, which is very close to the 5.33 duration.

22. Table 3.3 can be flipped and recalculated as follows:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Date | Year | Cash Payment | Discount Factor at 2% | PV | Fraction of Total Value |  | Year times Fraction of Value |
| Aug-09 | 0.5 | 5.63 | 0.990148 | 5.57 | 3.66% |  | 0.02 |
| Feb-10 | 1.0000 | 5.63 | 0.980 | 5.51 | 3.63% |  | 0.04 |
| Aug-10 | 1.5000 | 5.63 | 0.971 | 5.46 | 3.59% |  | 0.05 |
| Feb-11 | 2.0000 | 5.63 | 0.961 | 5.41 | 3.55% |  | 0.07 |
| Aug-11 | 2.5000 | 5.63 | 0.952 | 5.35 | 3.52% |  | 0.09 |
| Feb-12 | 3.0000 | 5.63 | 0.942 | 5.30 | 3.48% |  | 0.10 |
| Aug-12 | 3.5000 | 5.63 | 0.933 | 5.25 | 3.45% |  | 0.12 |
| Feb-13 | 4.0000 | 5.63 | 0.924 | 5.20 | 3.42% |  | 0.14 |
| Aug-13 | 4.5000 | 5.63 | 0.915 | 5.15 | 3.38% |  | 0.15 |
| Feb-14 | 5.0000 | 5.63 | 0.906 | 5.09 | 3.35% |  | 0.17 |
| Aug-14 | 5.5000 | 5.63 | 0.897 | 5.04 | 3.32% |  | 0.18 |
| Feb-15 | 6.0000 | 105.63 | 0.888 | 93.79 | 61.65% |  | 3.70 |
|  |  |  |  |  |  |  |  |
| TOTAL |  |  |  | 152.13 | 100.00% |  | 4.83 |

1. Decreasing the coupon payments to 8% of face boosts duration to 5.05:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Date | Year | Cash Payment | Discount Factor at 2% | PV | Fraction of Total Value |  | Year times Fraction of Value |
| Aug-09 | 0.5 | 4.00 | 0.990148 | 3.96 | 2.96% |  | 0.01 |
| Feb-10 | 1.0000 | 4.00 | 0.980 | 3.92 | 2.93% |  | 0.03 |
| Aug-10 | 1.5000 | 4.00 | 0.971 | 3.88 | 2.90% |  | 0.04 |
| Feb-11 | 2.0000 | 4.00 | 0.961 | 3.84 | 2.87% |  | 0.06 |
| Aug-11 | 2.5000 | 4.00 | 0.952 | 3.81 | 2.84% |  | 0.07 |
| Feb-12 | 3.0000 | 4.00 | 0.942 | 3.77 | 2.82% |  | 0.08 |
| Aug-12 | 3.5000 | 4.00 | 0.933 | 3.73 | 2.79% |  | 0.10 |
| Feb-13 | 4.0000 | 4.00 | 0.924 | 3.70 | 2.76% |  | 0.11 |
| Aug-13 | 4.5000 | 4.00 | 0.915 | 3.66 | 2.73% |  | 0.12 |
| Feb-14 | 5.0000 | 4.00 | 0.906 | 3.62 | 2.71% |  | 0.14 |
| Aug-14 | 5.5000 | 4.00 | 0.897 | 3.59 | 2.68% |  | 0.15 |
| Feb-15 | 6.0000 | 104.00 | 0.888 | 92.35 | 69.00% |  | 4.14 |
|  |  |  |  |  |  |  |  |
| TOTAL |  |  |  | 133.83 | 100.00% |  | 5.05 |

This makes sense as we are now receiving smaller payments early in the life of the bond.

1. Increasing the yield to 6% reduces duration to 4.70:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Date | Year | Cash Payment | Discount Factor at 2% | PV | Fraction of Total Value |  | Year times Fraction of Value |
| Aug-09 | 0.5 | 5.63 | 0.971 | 5.46 | 4.31% |  | 0.02 |
| Feb-10 | 1.0000 | 5.63 | 0.943 | 5.31 | 4.19% |  | 0.04 |
| Aug-10 | 1.5000 | 5.63 | 0.916 | 5.15 | 4.07% |  | 0.06 |
| Feb-11 | 2.0000 | 5.63 | 0.890 | 5.01 | 3.95% |  | 0.08 |
| Aug-11 | 2.5000 | 5.63 | 0.864 | 4.86 | 3.84% |  | 0.10 |
| Feb-12 | 3.0000 | 5.63 | 0.840 | 4.72 | 3.73% |  | 0.11 |
| Aug-12 | 3.5000 | 5.63 | 0.816 | 4.59 | 3.62% |  | 0.13 |
| Feb-13 | 4.0000 | 5.63 | 0.792 | 4.46 | 3.52% |  | 0.14 |
| Aug-13 | 4.5000 | 5.63 | 0.769 | 4.33 | 3.42% |  | 0.15 |
| Feb-14 | 5.0000 | 5.63 | 0.747 | 4.20 | 3.32% |  | 0.17 |
| Aug-14 | 5.5000 | 5.63 | 0.726 | 4.08 | 3.22% |  | 0.18 |
| Feb-15 | 6.0000 | 105.63 | 0.705 | 74.46 | 58.80% |  | 3.53 |
|  |  |  |  |  |  |  |  |
| TOTAL |  |  |  | 126.63 | 100.00% |  | 4.70 |

Payments at the end of the bonds life are discounted more heavily, resulting in a greater fraction of total value being paid early.

23. The duration of a perpetual bond is: [(1 + yield)/yield]

The duration of a perpetual bond with a yield of 5% is:

D5 = 1.05/0.05 = 21 years

The duration of a perpetual bond yielding 10% is:

D10 = 1.10/0.10 = 11 years

Because the duration of a zero-coupon bond is equal to its maturity, the 15-year zero-coupon bond has a duration of 15 years.

Thus, comparing the 5% perpetual bond and the zero-coupon bond, the 5% perpetual bond has the longer duration. Comparing the 10% perpetual bond and the 15 year zero, the zero has a longer duration.

24. Answers will differ. Generally, we would expect yield changes to have the greatest impact on long-maturity and low coupon bonds.

25. The calculations are shown in the table below:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |  | Bond Price (PV) | YTM (%) |
| Spot rates | 4.60% | 4.40% | 4.20% | 4% |  |  |  |
| Discount factors | 0.9560 | 0.9175 | 0.8839 | 0.8548 |  |  |  |
|  |  |  |  |  |  |  |  |
| Bond A (8% coupon): |  |  |  |  |  |  |  |
| Payment (Ct) | 80 | 1080 |  |  |  |  |  |
| PV(Ct) | 76.48 | 990.88 |  |  |  | 1067.37 | 4.407% |
|  |  |  |  |  |  |  |  |
| Bond B (11% coupon): |  |  |  |  |  |  |  |
| Payment (Ct) | 110 | 110 | 1110 |  |  |  |  |
| PV(Ct) | 105.16 | 100.92 | 981.11 |  |  | 1187.20 | 4.226% |
|  |  |  |  |  |  |  |  |
| Bond C (6% coupon): |  |  |  |  |  |  |  |
| Payment (Ct) | 60 | 60 | 60 | 1060 |  |  |  |
| PV(Ct) | 57.36 | 55.05 | 53.03 | 906.09 |  | 1071.54 | 4.028% |
|  |  |  |  |  |  |  |  |
| Bond D (strip): |  |  |  |  |  |  |  |
| Payment (Ct) | 0 | 0 | 0 | 1000 |  |  |  |
| PV(Ct) |  |  |  | 854.80 |  | 854.80 | 4.00% |

26. We will borrow $1000 at a five year loan rate of 2.5% and buy a 4-year strip paying 4%. We may not know what interest rates we will earn on the last year (4🡪5) but our $1000 will come due and we put it in under our mattress earning 0% if necessary to pay off the loan.

Let’s turn to present value calculations: As shown above, the cost of the strip is $854.80. We will receive proceeds from the 2.5% loan = 1000 / (1.025)^5 = $883.90. Pocket the difference of $29.10, smile, and repeat.

The minimum sensible value would set the discount factors used in year 5 equal to that of year 4, which would assume a 0% interest rate from year 4 to 5. We can solve for the interest rate where 1/ (1+r)5 = 0.8548, which is roughly 3.19%

27.

a. If the expectations theory of term structure is right, then we can determine the expected future one year spot rate (at t=3) as follows: investing $100 in a 3-year instrument at 4.2% gives us 100 \* (1+.042)^3 = 113.136. Investing $100 in a 4-year instrument at 4.0% gives us 100 \* (1+.04)^4 = 116.986. This reveals a one year spot rate from year 3 to 4 of (116.98 – 113.136) / 113.136 = 3.4%

b. If investing in long-term bonds carries additional risks, then the risk equivalent one year spot rate in year three would be even less (reflecting the fact that some risk premium must be built into this 3.4% spot rate).

28.

a. Your nominal return will be1.082 -1 = 16.64% over the two years. Your real return is (1.08/1.03) x (1.08/1.05) - 1 = 7.85%

b. With the TIPS, the real return will remain at 8% per year, or 16.64% over two years. The nominal return on the TIPS will equal (1.08x1.03) x (1.08x1.05) – 1 = 26.15%.

29. The bond price at a 5.41% yield is:



If the yield increases to 8.47%, the price would decline to:



30. Spreadsheet problem; answers will vary.

31. Arbitrage opportunities can be identified by finding situations where the implied forward rates or spot rates are different.

We begin with the shortest-term bond, Bond G, which has a two-year maturity. Since G is a zero-coupon bond, we determine the two-year spot rate directly by finding the yield for Bond G. The yield is 9.5 percent, so the implied two-year spot rate (r2) is 9.5 percent. Using the same approach for Bond A, we find that the three-year spot rate (r3) is 10.0 percent.

Next we use Bonds B and D to find the four-year spot rate. The following position in these bonds provides a cash payoff only in year four:

a long position in two of Bond B and a short position in Bond D.

Cash flows for this position are:

[(–2 × $842.30) + $980.57] = –$704.03 today

[(2 × $50) – $100] = $0 in years 1, 2 and 3

[(2 × $1050) – $1100] = $1000 in year 4

We determine the four-year spot rate from this position as follows:



r4 = 0.0917 = 9.17%

Next, we use r2, r3 and r4 with one of the four-year coupon bonds to determine r1. For Bond C:



r1 = 0.3867 = 38.67%

Now, in order to determine whether arbitrage opportunities exist, we use these spot rates to value the remaining two four-year bonds. This produces the following results: for Bond B, the present value is $854.55, and for Bond D, the present value is $1,005.07. Since neither of these values equals the current market price of the respective bonds, arbitrage opportunities exist. Similarly, the spot rates derived above produce the following values for the three-year bonds: $1,074.22 for Bond E and $912.77 for Bond F.

32. We begin with the definition of duration as applied to a bond with yield r and an annual payment of C in perpetuity



We first simplify by dividing both the numerator and the denominator by C:



The denominator is the present value of a perpetuity of $1 per year, which is equal to (1/r). To simplify the numerator, we first denote the numerator S and then divide S by (1 + r):



Note that this new quantity [S/(1 + r)] is equal to the square of denominator in the duration formula above, that is:



Therefore:



Thus, for a perpetual bond paying C dollars per year:



33. We begin with the definition of duration as applied to a common stock with yield r and dividends that grow at a constant rate g in perpetuity:



We first simplify by dividing each term by [C(1 + g)]:



The denominator is the present value of a growing perpetuity of $1 per year, which is equal to [1/(r - g)]. To simplify the numerator, we first denote the numerator S and then divide S by (1 + r):



Note that this new quantity [S/(1 + r)] is equal to the square of denominator in the duration formula above, that is:



Therefore:



Thus, for a perpetual bond paying C dollars per year:



34. a. We make use of the one-year Treasury bill information in order to determine the one-year spot rate as follows:



r1 = 0.0700 = 7.00%

The following position provides a cash payoff only in year two:

a long position in twenty-five two-year bonds and a short position in one one-year Treasury bill. Cash flows for this position are:

[(–25 × $94.92) + (1 × $93.46)] = –$2,279.54 today

[(25 × $4) – (1 × $100)] = $0 in year 1

(25 × $104) = $2,600 in year 2

We determine the two-year spot rate from this position as follows:



r2 = 0.0680 = 6.80%

The forward rate f2 is computed as follows:

f2 = [(1.0680)2/1.0700] – 1 = 0.0660 = 6.60%

The following position provides a cash payoff only in year three:

a long position in the three-year bond and a short position equal to (8/104) times a package consisting of a one-year Treasury bill and a two-year bond. Cash flows for this position are:

[(–1 × $103.64) + (8/104) × ($93.46 + $94.92)] = –$89.15 today

[(1 × $8) – (8/104) × ($100 + $4)] = $0 in year 1

[(1 × $8) – (8/104) × $104] = $0 in year 2

1 × $108 = $108 in year 3

We determine the three-year spot rate from this position as follows:



r3 = 0.0660 = 6.60%

The forward rate f3 is computed as follows:

f3 = [(1.0660)3/(1.0680)2] – 1 = 0.0620 = 6.20%

1. We make use of the spot and forward rates to calculate the price of the 4 percent coupon bond:



The actual price of the bond ($950) is significantly greater than the price deduced using the spot and forward rates embedded in the prices of the other bonds ($931.01). Hence, a profit opportunity exists. In order to take advantage of this opportunity, one should sell the 4 percent coupon bond short and purchase the 8 percent coupon bond.

35. a. Bond D allows us to calculate the four year spot rate by solving for the YTM using the 841.78 price of Bond D:



r4 = 4.4%

We can then set up the following three equations using the prices of bonds A, B, and C:

Using bond A: 

Using bond B: 

Using bond C: 

We know r4 = 4.4% so we can substitute that into the last equation. Now we have three equations and three unknowns and can solve this with variable substitution or linear programming to get r1 = 3.5%, r2 = 4%; r3 = 4.2%; r4 = 4.4%

b. We will want to invest in the underpriced C and borrow money at the current spot market rates to construct an offsetting position. For example, we might borrow $60 at the 1 year rate of 3.5%, 60 at the 2 year rate of 4%, 60 at the 3 year rate of 4.2% and 1060 at the 4 year rate of 4.4%. Of course the PV amount we will receive on these loans is $1058.76. Now we purchase the discounted bond C at $1040, use the proceeds of this bond to repay our loans as they come due. We can pocket the difference of $18.76, smile, and repeat.