Electoral Poaching and Party Identification

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Abstract

This paper studies electoral competition in a model of redistributive politics with deterministic voting and heterogeneous voter loyalties to political parties. We construct a natural measure of “party strength” based on the sizes and intensities of attachment of a party’s loyal voter segments and demonstrate how party behavior varies with the two parties’ strengths. In equilibrium, parties aggressively target or “poach” a portion of the opposition party’s loyal voters. Each party’s level of poaching is decreasing, and, consequently, the level of inequality in each party’s equilibrium redistribution schedule is increasing, in the opposition party’s strength. We also construct a measure of “party polarization” that is increasing in the sum and symmetry of the parties’ strengths, and find that aggregate inequality is increasing in party polarization.
1 Introduction

In the model of redistributive politics, political parties compete for representation in a legislature by simultaneously announcing how they will allocate a budget by making binding promises to each voter. Each voter votes for the party offering the highest level of utility, and each party’s payoff is its representation in the legislature, which under proportional representation is equal to the fraction of votes received by that party. Originally formulated by Myerson (1993), the model has served as a fundamental tool in the analysis of electoral competition. In recent years, the model has attracted renewed interest through its application to the study of the inequality created by political competition (Laslier and Picard (2002), Laslier (2002)), incentives for generating budget deficits (Lizzeri (2002)), inefficiency of public good provision (Lizzeri and Persico (2001, 2002)), and campaign spending regulation (Saluguet and Persico (2004)).

This paper extends the model of redistributive politics to allow for heterogeneous voter loyalties to political parties and shows that this has important implications for the nature of redistributive competition. Voters are distinguished by the party with which they identify, if any, and the intensity of their attachment, or “loyalty,” to that party. We construct a natural measure of “party strength” based on both the sizes and intensities of a party’s loyal voter segments and demonstrate how party behavior varies with the two parties’ strengths. We demonstrate that each party’s representation in the legislature is increasing (decreasing) in its own (opponent’s) party strength. In addition, we find that the parties have an incentive to target or “poach” a subset of the opposition party’s loyal voters, in an effort to induce those
voters to vote against the opposition party. Each party’s level of poaching is decreasing, and, consequently, the level of inequality in each party’s equilibrium redistribution schedule is increasing, in the opposition party’s strength.

As is common in models of electoral competition, the policy implemented by the legislature is assumed to be a probabilistic compromise of the parties’ equilibrium redistribution schedules.\(^1\) The probability that a party’s schedule is adopted is proportional to the size of its legislative contingent. In this context, we find that there is a price of party loyalty. For a given distribution of voters’ attachments to the political parties, the equilibrium expected transfers and resulting expected utilities from the implemented policy are the highest for swing voters and are strictly decreasing in the intensity of attachment.\(^2\) Remarkably, this result is independent of the size of each of the segments of voters and the parties’ strengths.

Moreover, defining the “level of partisanship” as the sum of the party strengths, we find that partisanship preserving transformations of the electorate that increase the strength of party \(i\) at the expense of party \(−i\) result in party \(i\)’s loyal voters receiving higher expected utilities from the implemented policy and party \(−i\)’s loyal voters receiving lower expected utilities from the implemented policy. In addition, transformations of the electorate

\(^1\)This interpretation is due to Grossman and Helpman (1996). Probabilistic compromise can also be viewed as a system under which each party distributes a fraction of the budget, proportional to its representation in the legislature, according to its announced schedule. This approach is taken in Myerson (1993).

\(^2\)For expected transfers this result holds regardless of party affiliation, and for expected utilities it holds within each party.
that hold constant the difference in party strengths while increasing the level of partisanship result in all segments receiving higher expected transfers and utilities from the implemented policy.

We also develop a measure of “party polarization” that is increasing in the sum and symmetry of the parties’ strengths and show that aggregate inequality is increasing in party polarization. In particular, partisanship preserving transformations of the electorate that increase the difference in the parties’ strengths increase aggregate inequality, and, for a given level of partisanship, aggregate inequality is maximized when the parties are of equal strength. In addition, holding constant the difference in the parties’ strengths, aggregate inequality increases as the level of partisanship increases. That is, the level of partisanship and the level of symmetry in the parties’ strengths generate inequality.

Two related papers are Laslier (2002) and Dixit and Londregan (1996). Laslier (2002) examines the issue of tyranny of the majority\(^3\) in a model of redistributive politics with a segmented homogeneous electorate and intra-segment homogeneity in the parties’ redistribution schedules. That paper finds that as long as there does not exist a segment that contains over half of the voters there is no tyranny of the majority, but if any segment contains more than half of the voters the parties use the entire budget on that segment. Our model extends the Laslier model by allowing for a segmented heterogeneous electorate and intra-segment heterogeneity in the parties’ redistribution schedules. We find that the outcome exhibiting tyranny of the

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\(^3\)Tocqueville describes tyranny of the majority as the situation where “the interests of the many are to be preferred to the few.”
majority is an artifact of the assumption of intra-segment homogeneity in the parties’ redistribution schedules. With intra-segment heterogeneity in the parties’ redistribution schedules there is no tyranny of the majority.

Dixit and Londregan (1996, henceforth D-L) is perhaps closest to our paper in its focus. Each features voters who derive utility from both redistribution and party identification. Unlike our model, D-L assume both intra- and inter-segment heterogeneity of voters. In addition, D-L impose intra-segment homogeneity of the parties’ redistribution schedules, which makes it impossible to directly target voters by their intensity of party attachment. Our model extends that paper by allowing parties to directly target voters.

In section 2 we present the model and characterize the unique equilibrium of the game of redistributive politics with party identification. Section 3 examines the expected transfers and utilities from each party’s equilibrium redistributive schedule and from the implemented policy. Section 4 examines how the aggregate inequality arising from each party’s redistribution schedule and from the implemented policy varies with changes in the distribution of voters’ attachments to the two political parties. Section 4 also addresses the relationship between aggregate inequality and party polarization. Section 5 concludes.

2 The Model

Political Parties and the Legislature

4We allow for third degree transfer discrimination akin to third degree price discrimination in the industrial organization literature.
Our model extends Myerson’s (1993) two-party model of redistributive competition by including heterogeneous voter loyalties to political parties.

Two parties, A and B, make simultaneous offers to each of a continuum of voters of unit measure. Each party’s payoff is its representation in the legislature. With two parties and proportional representation, this objective is equivalent to maximizing vote share. All offers must be nonnegative and each party has a budget of 1, which corresponds to 1 unit of a homogeneous good per voter. Parties are assumed to have complete information regarding the party preferences of all voters. While this is a stylized assumption, this is not an unreasonable benchmark given the high level of organization of modern political parties.\footnote{See for example PBS (2004) which discusses the high level of information that national political parties have access to and use to target voters.}

As is commonly assumed in the literature on electoral competition, the legislature implements a policy that is a probabilistic compromise of the parties’ redistribution schedules.

**Definition D.1:** The implemented policy is the weighted sum of each party’s equilibrium redistribution schedule multiplied by its respective vote share.

**Voters**

Voters are distinguished by the party with which they identify, if any, and the intensity of their attachment to that party. In this paper, we consider only distributions of voters’ attachments to the political parties with support on a finite set of intensities of attachment. Let $\delta_i^j \in (0, 1)$ represent the number
of units of the homogeneous good that party $i$ must offer a loyal voter in its own loyal segment $j$ in order to make that voter indifferent between the two parties when party $-i$ offers one unit of the homogeneous good.\footnote{This type of effectiveness advantage originates, to the best of our knowledge, with Lein (1990) and is frequently used in the literature on unfair contests (see for instance: Clark and Riis (2000), Konrad (2002), and Sahuguet and Persico (2004)).} Thus, the utility that each loyal voter in party $i$’s segment $j$ receives from an offer of $x^A$ from party $A$ is

$$
u_j^i (x^A) = \begin{cases} 
    x^A & \text{if } i = B \\
    \frac{x^A}{\delta_j^A} & \text{if } i = A
\end{cases}$$

Define $\alpha_j^i = 1 - \delta_j^i$ to be the intensity of attachment of party $i$’s loyal voter segment $j$. Party $A$’s loyal voters have a finite number, $n_A$, of different intensities of attachment. Let $A$ be the set of all indices of intensity of attachment for voters loyal to party $A$. Each index of intensity $j \in A$ corresponds to a segment of voters with intensity of attachment $\alpha_j^A$ and measure $m_j > 0$. Likewise, party $B$’s loyal voters have a finite number, $n_B$, of different intensities of attachment. Let $B$ be the set of all indices of intensity of attachment for voters loyal to party $B$, where $A$ and $B$ are disjoint sets. Each index of intensity $k \in B$ corresponds to a segment of voters with intensity of attachment $\alpha_k^B$ and measure $m_k > 0$. There are also swing voters who do not identify with either party. Let $S$ be the index for no attachment to either party. The measure of swing voters is $m_S = 1 - \sum_{j \in A \cup B} m_j$. To summarize $\left\{ \{m_j, \alpha_j^A\}_{j \in A}, \{m_k, \alpha_k^B\}_{k \in B} \right\}$ is a feasible distribution of voters’ attachments to the political parties if $n_A$ and $n_B$ are finite, $\sum_{j \in A \cup S \cup B} m_j = 1$, and $m_j > 0$ for all $j \in A \cup B$.

Voters cast their votes using a standard plurality scoring rule. Letting
$s^i_A : \mathbb{R}_+^2 \rightarrow \{0,1\}$ designate voter $i$’s vote for party $A$ and $x^A_i$, $x^B_i$ denote party $A$’s and party $B$’s offers to voter $i$ respectively, $s^i_A (x^A_i, x^B_i)$ equals 0 if

1. voter $i$ is independent and $x^A_i < x^B_i$,

2. voter $i$ is loyal to party $A$ with intensity $a^j_A$ and $\frac{x^A_i}{1-a^j_A} < x^B_i$, or

3. voter $i$ is loyal to party $B$ with intensity $a^k_B$ and $x^A_i < \frac{x^B_i}{1-a^k_B}$.

Similarly $s^i_A (x^A_i, x^B_i)$ equals +1 if

1. voter $i$ is independent and $x^A_i > x^B_i$,

2. voter $i$ is loyal to party $B$ with intensity $a^k_B$ and $x^A_i > \frac{x^B_i}{1-a^k_B}$, or

3. voter $i$ is loyal to party $A$ with intensity $a^j_A$ and $\frac{x^A_i}{1-a^j_A} > x^B_i$.

For party $B$, $s^i_B (x^A_i, x^B_i)$ is defined as $s^i_B (x^A_i, x^B_i) = 1 - s^i_A (x^A_i, x^B_i)$. Thus a voter who is independent votes for the party that makes him the higher offer, while a loyal voter requires a proportionally higher offer from the rival party in order to induce him to cross over. Representation in the legislature is allocated proportionally. Thus, we normalize each party’s representation in the legislature to be equal to the fraction of the votes received by that party.

One simple yet important summary statistic of a party’s distribution of loyal voters is the sum across segments of each segment’s intensity of attachment weighted by the measure of the set of voters attached to the segment.

**Definition D.2:** The strength of party $A$ is denoted by $\sigma_A \equiv \sum_{j \in A} m_j a^j_A$. The strength of party $B$ is denoted by $\sigma_B \equiv \sum_{k \in B} m_k a^k_B$. 

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Several properties of this summary statistic should be noted. First, holding constant the size of each of a party’s loyal segments, the party’s strength is strictly increasing in the intensity of the attachment of any of these segments. Second, holding constant the intensity of attachment of each of its loyal segments, the party’s strength is strictly increasing in the size of each of these segments. Finally, holding constant a party’s size, the party’s strength is strictly increasing as loyal voters shift from weaker intensities of attachment to stronger intensities of attachment.

Given the parties’ strengths, $\sigma_A$ and $\sigma_B$, it is useful to derive two simple measures for the distribution of voters’ attachments to the political parties.

**Definition D.3:** The level of partisanship is the sum of the party strengths which is denoted by $\sigma \equiv \sigma_A + \sigma_B$.

**Definition D.4:** The effective strength of party $i$ is denoted by $\hat{\sigma}_i \equiv \sigma_i - \sigma_{-i}$.

The level of partisanship is the sum across the entire electorate of each segment’s intensity of attachment, to either party, weighted by the measure of the set of voters attached to the segment. The properties of the level of partisanship are similar to those of party strengths. Holding constant the size of each loyal segment, the level of partisanship is strictly increasing in the intensity of the attachment of each segment. In addition, the level of partisanship is strictly increasing as loyal voters shift from weaker intensities of attachment to stronger intensities of attachment and/or as swing voters become affiliated with a political party. The effective strength of party $i$
measures the asymmetry between party $i$ and party $-i$. If the parties have symmetric strengths then each party has an effective strength of 0.

**Redistributive Competition**

A strategy, which we label a *redistributive schedule*, for party $i$ is a set of cumulative distribution functions,\(^7\) \(\{F^j_i\} \_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}}\) one distribution function for each segment $j \in \mathcal{A}$ of voters loyal to party $A$, the segment of swing voters $\mathcal{S}$, and each segment $k \in \mathcal{B}$ of voters loyal to party $B$. As in Myerson (1993) each $F^j_i(x)$ denotes the fraction of voters in segment $j$ whom party $i$ will offer less than $x$. The only restrictions that are placed on the set of feasible strategies is that each offer must be nonnegative and the set of cumulative distribution functions must satisfy the budget constraint:

$$\sum_{j \in \mathcal{A} \cup \mathcal{S} \cup \mathcal{B}} m_j \int_0^\infty x dF^j_i \leq 1 \quad (1)$$

*Redistributive competition* is the one-shot game, which we label

$$G \left( \left\{ \{m_j, a^j_\mathcal{A}\}_{j \in \mathcal{A}}, \{m_k, a^k_\mathcal{B}\}_{k \in \mathcal{B}} \right\} \right)$$

in which parties compete for representation in the legislature by simultaneously announcing redistributive schedules, subject to a budget constraint.

\(^7\)In this case the focus is on the distributions within each segment (marginal distributions) rather than an n-variate joint distribution. As discussed in the appendix, an n-variate joint distribution is trivial to obtain and adds nothing to the problem analyzed here.
Optimal Strategies

The following theorem characterizes the equilibrium of the redistributive
competition game.

**Theorem 1:** The unique Nash equilibrium of the redistributive
competition game $G \left( \left\{ \{m_j, a^j_A\}_{j \in A}, \{m_k, a^k_B\}_{k \in B} \right\} \right)$ is for each
party $i$ to choose offers according to the following distributions.

For party $A$

\[
\forall j \in A \quad F^j_A(x) = \frac{x}{z(1-a^j_A)} \quad x \in [0, z (1-a^j_A)] \\
F^S_A(x) = \frac{x}{z} \quad x \in [0, z]
\]

\[
\forall k \in B \quad F^k_A(x) = a^k_B + (1 - a^k_B) \frac{x}{z} \quad x \in [0, z].
\]

Similarly for party $B$

\[
\forall k \in B \quad F^k_B(x) = \frac{x}{z(1-a^k_B)} \quad x \in [0, z (1-a^k_B)] \\
F^S_B(x) = \frac{x}{z} \quad x \in [0, z]
\]

\[
\forall j \in A \quad F^j_B(x) = a^j_A + (1 - a^j_A) \frac{x}{z} \quad x \in [0, z].
\]

where \( z = \frac{2}{1-\sigma} = \frac{2}{1-\sigma_A - \sigma_B} \). In equilibrium party $A$’s share of
the vote is \( \frac{1+\hat{\sigma}_A}{2} = \frac{1+\sigma_A - \sigma_B}{2} \), and party $B$’s share of the vote is
\( \frac{1+\hat{\sigma}_B}{2} = \frac{1+\sigma_B - \sigma_A}{2} \).

**Proof:** We begin by showing that this is an equilibrium. First,
this is a feasible strategy since:

\[
\sum_{j \in A \cup S \cup B} m_j \int_0^\infty x dF^j_i = 1
\]
Then given that party $B$ is following the equilibrium strategy, the vote share $\pi_A(\cdot)$ for party $A$, when it chooses to provide transfers according to an arbitrary strategy $\{\bar{F}_j\}_{j \in A \cup S \cup B}$ is:

$$\pi_A \left( \{\bar{F}_j, F_j\}_{j \in A \cup S \cup B} \right) = \sum_{j \in A} m_j \int_0^\infty F_B^j \left( \frac{x}{1 - a_A^j} \right) d\bar{F}_j^A(x)$$

$$+ m_S \int_0^\infty F_S^B(x) d\bar{F}_A^S(x)$$

$$+ \sum_{k \in B} m_k \int_0^\infty F_k^B(x\delta_B^{k}) d\bar{F}_k^A(x)$$

Since it is never a best response for party $A$ to provide offers outside the support of party $B$’s offers, we have:

$$\pi_A \left( \{\bar{F}_j, F_j\}_{j \in A \cup S \cup B} \right) = \frac{1}{z} \sum_{j \in A} m_j \int_0^{x(1 - a_A^j)} xd\bar{F}_j^A(x)$$

$$+ \sum_{j \in A} m_j a_A^j + \frac{m_S}{z} \int_0^\infty xd\bar{F}_A^S(x)$$

$$+ \frac{1}{z} \sum_{k \in B} m_k \int_0^\infty xd\bar{F}_k^A(x)$$

But from the budget constraint given in equation (1) it follows that

$$\pi_A \left( \{\bar{F}_j, F_j\}_{j \in A \cup S \cup B} \right) \leq \frac{1}{z} + \sum_{j \in A} m_j a_A^j = \frac{1 + \sigma_A - \sigma_B}{2}$$

which holds with equality if $\{\bar{F}_j\}_{j \in A \cup S \cup B}$ is the equilibrium strategy. Thus party $A$’s vote share cannot be increased by deviating to another strategy. The argument for party $B$ is symmetric.

In the appendix, the strategic equivalence between two-party games of redistributive politics with segmented voters and independent simultaneous two-bidder all-pay auctions is established.

The proof of uniqueness then follows from the arguments appearing in Baye, Kovenock and de Vries (1996). Q.E.D.
The following example illustrates the key features of Theorem 1.

**Example:** Assume that there are only two types of voters: voters loyal to party $A$ and voters loyal to party $B$. Let $m_A = \frac{1}{3}$, $a_A = \frac{1}{2}$, $m_B = \frac{2}{3}$, and $a_B = \frac{3}{4}$. Party $A$’s and party $B$’s strengths are $\sigma_A = \frac{1}{3} \left( \frac{1}{2} \right) = \frac{1}{6}$ and $\sigma_B = \frac{2}{3} \left( \frac{3}{4} \right) = \frac{1}{2}$, respectively. Party $A$’s and party $B$’s equilibrium vote shares are $\frac{1+\hat{\sigma}_A}{2} = \frac{1+\sigma_A-\sigma_B}{2} = \frac{1}{3}$ and $\frac{1+\hat{\sigma}_B}{2} = \frac{1+\sigma_B-\sigma_A}{2} = \frac{2}{3}$, respectively. The transfers and resulting utilities from the unique equilibrium redistribution schedules given by Theorem 1 are shown in Figure 1 below. As party $-i$’s loyal voters’ intensity of attachment, $a_{-i}$, increases, party $i$’s poaching of $-i$’s loyal voters decreases since $F_i^{-i}(0) = a_{-i}$ increases. This is represented graphically in Figure 1(a) and 1(b) as shift up of $F_i^{-i}(0)$.

[Insert Figure 1 here]

Note that, each party’s equilibrium vote share is increasing (decreasing) in its own (opponent’s) party strength. Party identification also creates an incentive for parties to target or poach the opposition party’s loyal voters, and the amount of poaching, which we measure as the proportion of the opposition party’s loyal voters that are targeted, is decreasing in the strength of the opposition party. As the strength of a party increases, the cost of poaching that party’s loyal voters increases and the amount of poaching by the opponent decreases as the opponent optimally targets more resources at their own loyal voters and the swing voters. As both parties become stronger.
the swing voters become more pivotal in the election. As a result both parties must devote more resources to the swing voters.

A similar poaching effect has been addressed in the industrial organization literature on brand loyalty and brand switching. For example, Fudenberg and Tirole (2000)\(^8\) examine a duopoly model of brand loyalty and brand switching where firms try to poach the competitor’s loyal consumers. The electoral poaching examined here differs from Fudenberg and Tirole (2000) in that the focus is on redistribution rather than short-term versus long-term contracts.

### 3 Equilibrium Transfers and Utilities

We now apply Theorem 1 to examine the equilibrium expected transfers and the resulting utilities of each party’s redistribution schedule and the implemented policy. In redistributive competition with heterogeneous voter loyalties, each candidate announces a distribution of offers for each segment of the electorate. In the discussion that follows we refer to a segment’s equilibrium expected transfer from a party’s redistribution schedule as the expectation of that party’s equilibrium distribution of offers on that segment. A segment’s equilibrium expected utility from a party’s redistribution schedule and the implemented policy, as well as the equilibrium expected transfer from the implemented policy are similarly defined.

Despite the fact that from Theorem 1 the parties’ equilibrium redistributive schedules differ in all segments of loyal voters, for each segment, the

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\(^8\) See also Lee (1997).
expected transfer from each party, and thus from the implemented policy, is the same. Furthermore, for a given distribution of voters’ attachments to the political parties, the expected transfers are highest for the swing segments and are strictly decreasing in the intensity of attachment. Thus, the most attached voters receive the lowest expected transfers and swing voters receive the highest expected transfers.

**Corollary 1:** Within any given voter segment, the expected transfers from the two parties are identical. For a given distribution of voters’ attachments to the political parties, the expected transfers are strictly decreasing in the intensity of attachment (regardless of party affiliation).

**Proof:** From Theorem 1 the swing voters equilibrium expected transfer from each party and from the implemented policy $E^S(\cdot)$ is

$$E^S \left( \left\{ \left\{ m_j, a^j_j \right\}_{j \in A}, \left\{ m_k, a^k_k \right\}_{k \in B} \right\} \right) = \frac{1}{1-\sigma}$$

Similarly, for each segment $j \in A$ of party $A$’s loyal voters the equilibrium expected transfer from each party and from the implemented policy $E^j(\cdot)$ is

$$E^j \left( \left\{ \left\{ m_j, a^j_A \right\}_{j \in A}, \left\{ m_k, a^k_B \right\}_{k \in B} \right\} \right) = \frac{1-a^j_A}{1-\sigma}$$

The argument for voters loyal to party $B$ is symmetric. Q.E.D.

The swing voter segment is the most contested segment since neither party has an advantage, and, thus, the equilibrium transfers are the highest in this segment. The presence of voter loyalties to the political parties creates
an incentive for the parties to try and poach a portion of the opposition party’s loyal voters. However, this poaching incentive is strictly decreasing in the intensity of attachment since poaching becomes more costly as the intensity of attachment increases. Since the amount of poaching decreases in the intensity of attachment, the affiliated party optimally diverts resources away from its most attached loyal voter segments to the other segments. This result is independent of the measures of the segments and the parties’ strengths.

Given fixed intensities of attachment and holding partisanship, \( \sigma = \sigma_A + \sigma_B \), constant, changes in the distribution of the voters’ attachments to the parties result in the same expected transfers for those who do not change preferences. Furthermore, for fixed intensities of attachment to the political parties, partisanship increasing transformations of the electorate increase each segment’s expected transfer from each party and from the implemented policy.

**Corollary 2:** Given fixed intensities of attachment to the parties, partisanship preserving transformations of the electorate have no effect on the segments’ expected transfers, and partisanship increasing transformations of the electorate increase each segment’s expected transfers.

**Proof:** The proof follows directly from the segments’ expected transfers given in corollary 1. Q.E.D.

The intuition for this is that, given fixed intensities of attachment, partisanship preserving transformations of the electorate result in the same amount of
poaching in the aggregate. Regardless of which party gains and which party loses in a partisanship preserving transformation of the electorate the same amount of poaching and the same amount of defense from poaching takes place. Similarly, partisanship increasing transformations of the electorate result in a lower amount of poaching in the aggregate as the cost of poaching has increased. As the cost of poaching increases the expected transfers also increase.

While these corollaries are stated in terms of the transfers, the resulting utilities display similar properties. One noticeable change is that for loyal voters the expected utilities from the affiliated party’s redistribution schedule are higher than the expected utilities from the opposition party’s redistribution schedule. In fact, the expected utility that each segment of loyal voters receive from the affiliated party’s redistribution schedule is equal to the expected utility that the swing voters receive from either party’s redistribution schedule. However, for the resulting utilities, it is still true that, for a given distribution of voters’ attachments to the political parties, the expected utility that each segment of loyal voters’ receives from the opposition party’s redistribution schedule is strictly decreasing in the intensity of attachment.

**Corollary 3:** For all loyal voter segments, the expected utility from the affiliated party’s redistribution schedule is equal to the expected utility that the swing voters receive from either party’s redistribution schedule. For a given distribution of voters’ attachments to the political parties, loyal voters’ expected utilities from the opposition party’s redistribution schedule are strictly decreasing in the intensity of attachment.
Proof: From Theorem 1, for each segment $j \in A$ of party $A$’s loyal voters the equilibrium expected transfer from the affiliated party $EU_A^j(\cdot)$ is

$$EU_A^j \left( \left\{ \{m_j, a_A^j\}_{j \in A}, \{m_k, a_B^k\}_{k \in B} \right\} \right) = \frac{1}{1-a_A^j} E^j = \frac{1}{1-\sigma}$$

which, from corollary 1, is equal to the expected utility for swing voters, $EU^S = E^S$. The argument for voters loyal to party $B$ is symmetric.

The second part of the corollary follows from the fact that for each segment $j \in A$ of party $A$’s loyal voters the equilibrium expected utility from the opposition party $EU_B^j(\cdot)$ is

$$EU_B^j \left( \left\{ \{m_j, a_A^j\}_{j \in A}, \{m_k, a_B^k\}_{k \in B} \right\} \right) = E^j = \frac{1-a_A^j}{1-\sigma}$$

The argument for voters loyal to party $B$ is symmetric. Q.E.D.

The connection between loyal voters’ utilities from the affiliated party’s redistribution schedule and the utilities of the swing voters goes further than just equivalent expectations. Within each segment of loyal voters the distribution of utilities resulting from the affiliated party’s redistribution schedule is identical to the distribution of utilities that the swing voters receive from both parties’ redistribution schedules. However, loyal voters’ expected utilities from the opposition party’s redistribution schedule are strictly decreasing in the intensity of attachment since the amount of poaching on a segment is strictly decreasing in the intensity of attachment.

There is also an analogous form of Corollary 2 for the expected utilities. Given fixed intensities of attachment, partisanship preserving transformations of the electorate do not change the expected utilities from the parties’
redistribution schedules for those who do not change preferences. Furthermore, for fixed intensities of attachment to the political parties, partisanship increasing transformations of the electorate increase each segment’s expected utility from each party.

**Corollary 4:** Given fixed intensities of attachment to the parties, partisanship preserving transformations of the electorate have no effect on each segment’s expected utility from each party’s redistribution schedule, and partisanship increasing transformations of the electorate increase each segment’s expected utility from each party’s redistribution schedule.

**Proof:** The proof follows directly from the segments’ expected transfers given in corollary 3. Q.E.D.

To measure changes in the expected utility from the implemented policy, we must take into account changes in both the level of partisanship and in the parties’ effective strengths. In particular, for fixed intensities of attachment, partisanship preserving transformations of the electorate that increase the strength of party $i$ increase party $i$’s loyal voters’ expected utilities from the implemented policy and decrease party $-i$’s loyal voters’ expected utilities from the implemented policy. Conversely, effective party strength preserving transformation of the electorate that increase the level of partisanship increase all voters expected utilities.

**Corollary 5:** Given fixed intensities of attachment to the parties, partisanship preserving transformations of the electorate that increase the strength of party $i$ increase party $i$’s loyal voters’ ex-
pected utilities from the implemented policy and decrease party
−i’s loyal voters’ expected utilities from the implemented policy.
In addition, effective party strength preserving transformations
of the electorate that increase the level of partisanship increase
all voters expected utilities.

Proof: From Theorem 1, for each segment \( j \in A \) of party \( A \)’s
loyal voters the equilibrium expected transfer from the imple-
mented policy \( \text{EU}^j (\cdot) \) is

\[
\text{EU}^j \left( \left\{ \{ m_j, a^j_A \}_{j \in A}, \{ m_k, a^k_B \}_{k \in B} \right\} \right) = \\
\left( \frac{1+\hat{\sigma}_A}{2} \right) \left( \frac{1}{1-\sigma} \right) + \left( \frac{1-\hat{\sigma}_B}{2} \right) \left( \frac{1-a^j_A}{1-\sigma} \right)
\]

which is increasing in \( \hat{\sigma}_A \) and thus decreasing in \( \hat{\sigma}_B = -\hat{\sigma}_A \). The
argument for voters loyal to party \( B \) is symmetric.

The second part of the corollary follows from the fact that for
each segment \( j \in A \)

\[
\frac{\partial \text{EU}^j}{\partial \sigma} > 0
\]

The argument for swing voters and voters loyal to party \( B \) follows
directly. Q.E.D.

4 Inequality and Party Polarization

We now apply Theorem 1 to examine how voter loyalty to political parties
affects the inequality arising from the equilibrium redistribution schedules.
In this section we will focus on inequality in the distribution of utilities.
It is also instructive to examine inequality in the distribution of transfers.
Remarkably, the inequality in the distribution of transfers is considerably more complex than the inequality in the distribution of utilities. Corollary 6 summarizes our results on the aggregate inequality, in the distribution of utilities, arising from each party’s equilibrium redistribution schedule as measured by the Gini-coefficient of inequality.

**Corollary 6:** For each party \( i = A, B \), the aggregate inequality (as measured by the Gini-coefficient of inequality) arising from the party’s equilibrium redistribution schedule is increasing in the opposing party’s strength. More precisely, the Gini-coefficient of party \( i \)'s equilibrium redistribution schedule is \( C_i = \frac{1}{3} + \frac{2\sigma_i}{3} \), \( i = A, B \).

**Proof:** From Theorem 1, the measure of the set of voters who receive a utility level from party A’s equilibrium redistribution schedule that is less than or equal to \( x \) is

\[
\tilde{F}_A(x) = \sum_{k \in B} m_k a_B^k + \frac{x}{z} \left( \sum_{k \in B} m_k (1 - a_B^k) + \sum_{j \in A \cup S} m_j \right)
\]

for \( x \in [0, z] \). Simplifying, \( \tilde{F}_A(x) = \sigma_B + \frac{x}{z} (1 - \sigma_B) \) for \( x \in [0, z] \).

By definition the Lorenz curve for \( \tilde{F}_A \) is

\[
L_A(y) = \frac{\int_{0}^{y} \tilde{F}_A^{-1}(x) \, dx}{\int_{0}^{1} \tilde{F}_A^{-1}(x) \, dx}
\]

for all \( y \in [0, 1] \). Simplifying,

\[
L_A(y) = \begin{cases} 
0 & \text{if } y \in [0, \sigma_B] \\
\frac{(y - \sigma_B)^2}{(1 - \sigma_B)} & \text{if } y \in (\sigma_B, 1]
\end{cases}
\]

For the case when inequality is measured in terms of transfers instead of utilities, counterexamples to corollaries 6-8 are available from the authors.
By definition the Gini-coefficient for \( \tilde{F}_A \) is

\[
C_A \left( \left\{ \{m_j, \delta^i_A\} \right\}_{j \in A}, \left\{ \{m_k, \delta^i_B\} \right\}_{k \in B} \right) = 1 - 2 \int_{\sigma_B}^1 L_A(x) \, dx.
\]

Simplifying we have \( C_A = \frac{1}{3} + \frac{2\sigma_B}{3} \). It follows that \( \frac{\partial C_A}{\partial \sigma_B} > 0 \). A similar argument establishes the property for party \( B \)'s equilibrium redistribution schedule. Q.E.D.

Each party \( i \) has an incentive to poach a different fraction of the voters from each of party \( -i \)'s loyal segments. As the intensity of attachment of a given segment of \( -i \)'s voters increases, the level of poaching (the measure of the set of voters receiving a non-zero offer) from that segment decreases. As a result, the aggregate inequality in party \( i \)'s equilibrium redistribution schedule increases.

More generally, as Corollary 6 states, any change in the distribution of voters’ attachments to the political parties that leads to an increase in the strength of party \( -i \), results in an increase in the aggregate inequality of party \( i \)'s equilibrium redistribution schedule. Moreover, poaching by party \( i \) decreases in the sense that a larger portion of party \( -i \)'s loyal voters receive a transfer of 0 from party \( i \).

The implications of these results for the aggregate inequality of the implemented policy are examined in the following corollary.

**Corollary 7:** Partisanship preserving transformations of the electorate that increase the symmetry in the parties’ strengths increase the aggregate inequality in the implemented policy. Moreover, for a given level of partisanship, \( \sigma \), the aggregate inequality arising from the implemented policy is maximized when the
parties are of equal strength, $\sigma_A = \sigma_B$. Conversely, effective party strength preserving transformations of the electorate that increase the level of partisanship increase the aggregate inequality in the implemented policy.

**Proof:** From Corollary 3, the aggregate inequality arising from the implemented policy is

$$I(\sigma_A, \sigma_B) = \left(\frac{1+\hat{\sigma}_A}{2}\right) \left(\frac{1+3\sigma_B}{3}\right) + \left(\frac{1-\hat{\sigma}_A}{2}\right) \left(\frac{1+3\sigma_A}{3}\right)$$

Simplifying we have

$$I(\sigma_A, \sigma_B) = \frac{1}{3} + \frac{\sigma - (\hat{\sigma}_A)^2}{3}.$$  

The first and third parts of the corollary follow directly. For the second part of the corollary not that for a given level of partisanship, $\sigma$, $I(\sigma_A, \sigma_B)$ is clearly maximized when $\hat{\sigma}_A = 0$ or $\sigma_A = \sigma_B = \frac{\sigma}{2}$. Q.E.D.

Hence, for a given level of partisanship, symmetry in party strength generates inequality. Similarly, for given levels of effective party strengths, partisanship generates inequality.

Our results on party strength and inequality are closely related to issues arising in the literature on polarization.\textsuperscript{10} Although much of this literature deals with the distribution of income, its tenets can be adapted to our context of redistributive politics. An interesting question that arises is whether there is a simple measure defined over the given parameters, one that might be labeled “party polarization,” that has the property that the aggregate inequality derived from the implemented policy is increasing in the measure.

It turns out that the answer is yes. Indeed, we base this measure solely on the party strengths. Setting

\[ P(\sigma_A, \sigma_B) = \sigma - (\hat{\sigma}_A)^2 = \sigma - (\hat{\sigma}_B)^2 \]

it is easily demonstrated that aggregate inequality arising from the implemented policy is increasing in \( P(\cdot, \cdot) \).

**Corollary 8:** The aggregate inequality arising from the implemented policy is increasing in the party polarization measure \( P(\cdot, \cdot) \).

The level curves of the party polarization measure and the expected inequality are shown in Figure 2 below.

[Insert Figure 2 here]

Several properties of these level curves should be mentioned. First a level of partisanship defines a ‘budget’ line over possible combinations of party strengths. Thus, the properties of expected inequality from the implemented policy addressed in Corollary 7 can be seen graphically in Figure 2. Second, given that the parties have symmetric strengths, an increase in either party’s strength increases polarization and thus expected inequality. That is

\[ \frac{\partial I}{\partial \sigma_i} \bigg|_{\hat{\sigma}_A=0} > 0. \]

Furthermore, given that \( \sigma_i < \frac{1}{2} + \sigma_{-i} \), an increase in party \( i \)’s strength increases the inequality arising from the implemented policy. That is

\[ \frac{\partial I}{\partial \sigma_i} \bigg|_{\sigma_i < \frac{1}{2} + \sigma_{-i}} > 0. \]
5 Conclusion

This paper extends Myerson’s (1993) model of redistributive politics to allow for heterogeneous voter loyalties to political parties. Parties distinguish voters by the party with which they identify, if any, and the intensity of their attachment, or “loyalty,” to that party. We find that voters pay a price for party loyalty. For a given distribution of voters’ attachments to the political parties, in the implemented policy, the segment of swing voters has the highest expected transfer and expected utility, and the expected transfers and utilities for loyal voter segments are strictly decreasing in the intensity of attachment. Using our measure of “party strength,” based on both the sizes and intensities of attachment of a party’s loyal voter segments, we demonstrate that each party’s representation in the legislature is increasing (decreasing) in its own (opponent’s) party strength. In addition, parties poach a subset of the opposition party’s loyal voters, in an effort to induce those voters to vote against the opposition party. Each party’s level of poaching is decreasing, and, consequently, the level of inequality in each party’s equilibrium redistribution schedule is increasing, in the opposition party’s strength.

We also develop a measure of “party polarization” that is increasing in the sum and symmetry of the party strengths, and find that aggregate inequality is increasing in party polarization. That is, the level of partisanship and the level of symmetry in the parties’ strengths generate inequality.

There are several directions for future research based on the model of redistributive politics with segmented loyal voters. This model can be used to shed light on some of the topics previously studied in the redistributive
politics literature, such as candidate valence issues. In addition, this paper’s focus on identifiable voter segments is quite applicable for the study of transfers targeted at geographical regions or other identifiable characteristics.

References


Appendix

Sahuguet and Persico (2004) establish an equivalence between the two-party model of redistributive politics and an appropriately chosen two-bidder all-pay auction. We now extend this result to establish an equivalence between the two-party model of redistributive politics with segmented loyal voters and an appropriately chosen set of two-bidder independent simultaneous all-pay auctions.

We begin by reviewing the characterization of \( n \) simultaneous two-bidder all-pay auctions with complete information. Let \( F_j^i \) represent bidder \( i \)'s distribution of bids for auction \( j \), and \( v_j^i \) represent the value of auction \( j \) for bidder \( i \). Each bidder \( i \)'s problem is

\[
\max \left\{ F_j^i \right\}_{j=1}^{n} \sum_{j=1}^{n} \int_{0}^{\infty} \left[ v_j^i F_j^i (x) - x \right] dF_j^i
\]

Since each auction is independent, the unique equilibrium is for each bidder to choose \( F_j^i \) as if auction \( j \) was the only auction. The case of a single all-pay auction with complete information is studied by Baye, Kovenock, de Vries (1996). Thus, for each auction \( j \) and bidder \( i \) we have the following three cases

- If \( v_j^i > v_j^i \), then \( F_j^i (x) = \frac{x}{v_j^i} \), for \( x \in [0, v_j^i] \)
- If \( v_j^i = v_j^i \), then \( F_j^i (x) = \frac{x}{v_j^i} \), for \( x \in [0, v_j^i] \)
- If \( v_j^i < v_j^i \), then \( F_j^i (x) = \left( \frac{v_j^i - v_j^i}{v_j^i} \right) + \frac{x}{v_j^i} \), for \( x \in [0, v_j^i] \)

In addition, without a binding cap on bids, there is no reason to construct an \( n \)-variate distribution function from these marginal distributions.\(^{11}\)

\(^{11}\)Without a binding cap on bids, it is trivial to construct an \( n \)-variate distribution
Now consider two-party redistributive competition with segmented loyal voters, and assume that the parties face the budget constraint

$$\sum_{j \in A \cup S \cup B} m_j \int_0^\infty x dF_j^i \leq 1,$$

where $F_i^j$ represents party $i$'s distribution of offers for voters in segment $j$ and $m_j > 0$ is the measure of voters in segment $j$ such that $\sum_{j \in A \cup S \cup B} m_j = 1$. In the discussion that follows the notation for the intensity of loyal voter attachment is modified in the following way: for each segment $j \in A \cup S \cup B$ if $j = S$, the swing segment, or $j \in B$, one of party $B$’s loyal voter segments, then $a^i_A = 0$, thus $\delta^i_A = 1$, and the same holds for $a^k_B$ if $k \in A \cup S$. Each party $i$’s problem is

$$\max \{ \{F_i^j\} \}_{j \in A \cup S \cup B} \sum_{j \in A \cup S \cup B} m_j \int_0^\infty F_j^i \left( \frac{x \delta^{-1}_i}{\delta^i_j} \right) dF_j^i$$

subject to the budget constraint $\sum_{j \in A \cup S \cup B} m_j \int_0^\infty x dF_j^i (x) \leq 1$. The associated Lagrangian is

$$\max \{ \{F_i^j\} \}_{j \in A \cup S \cup B} \sum_{j \in A \cup S \cup B} \left[ m_j \lambda_i \int_0^\infty \left[ \frac{1}{\lambda_i} F_j^i \left( \frac{x \delta^{-1}_i}{\delta^i_j} \right) - x \right] dF_j^i (x) \right] + \lambda_i$$

We can now proceed to the proof of the equivalence between the two-party model of redistributive politics with segmented loyal voters and an appropriately chosen set of two-bidder independent simultaneous all-pay auctions. In the discussion that follows, $\bar{s}_j$ and $\underline{s}_j$ are the upper and lower bounds of candidate $i$’s distribution of offers in segment $j$.

since any n-variate copula is sufficient. Given the Fréchet-Hoeffding bounds for n-variate copulas, the range of sufficient n-variate copulas is quite large. For this reason the n-variate joint distribution adds nothing to the problem analyzed here. See Nelson (1999) for an introduction to copulas.
Theorem 2: For each feasible distribution of voters’ attachments to the political parties, there exists a one-to-one correspondence between the equilibria of the two-party model of redistributive politics with segmented loyal voters and the equilibria of a unique set of appropriately chosen two-bidder independent simultaneous all-pay auctions.

Proof: The proof, which is contained in the following lemmas, is instructive in that it establishes the uniqueness of the equilibrium given in Theorem 1.

The first three lemmas follow from lines drawn by Baye, Kovenock, and de Vries (1996).

Lemma 1: For each \( j \in A \cup S \cup B \), \( \frac{s^j_i \delta^j_i}{\delta^j_{-i}} = \tilde{s}^j_i \).

Lemma 2: In any equilibrium \( \{ F^j_i, F^j_{-i} \} \), \( j \in A \cup S \cup B \), no \( F^j_i \) can place an atom in the half open interval \( (0, \tilde{s}^j_i] \).

Lemma 3: For each \( j \in A \cup S \cup B \) and for each \( i \in \{ A, B \} \),
\[
\frac{1}{\lambda_i} F^j_{-i} \left( \frac{x \delta^j_{-i}}{\delta^j_i} \right) - x \text{ is constant } \forall x \in (0, \tilde{s}^j_i].
\]

The following lemma characterizes the relationship between \( \lambda_i \) and \( \lambda_{-i} \).

Lemma 4: In equilibrium \( \lambda_i = \lambda_{-i} \).

Proof: By way of contradiction suppose \( \lambda_i \neq \lambda_{-i} \). In any equilibrium each party must use their entire budget, thus
\[
\sum_{j \in A \cup S \cup B} m_j \int_0^{\tilde{s}^j_i} xdF^j_i(x) = \sum_{j \in A \cup S \cup B} m_j \int_0^{\tilde{s}^j_{-i}} xdF^j_{-i}(x) \tag{2}
\]
But, from lemmas 2 and 3, it follows that
\[ dF^j_i(x) = \lambda \frac{\delta^j_i}{\delta^j_i} dx \] (3)
for all \( x \in (0, \bar{s}^j_i) \), and
\[ dF'^j_i(x) = \lambda_i \frac{\delta^j_i}{\delta^j_i} dx \] (4)
for all \( x \in (0, \bar{s}'^j_i) \). Substituting equations 4 and 5 into equation 3, and applying lemma 1 we have
\[ \lambda_i \sum_{j \in A \cup S \cup B} m_j \int_0^{\bar{s}^j_i} x \frac{\delta^j_i}{\delta^j_i} dx = \lambda_i \sum_{j \in A \cup S \cup B} m_j \int_0^{\bar{s}'^j_i} x \frac{\delta^j_i}{\delta^j_i} dx \]
which is a contradiction since
\[ \sum_{j \in A \cup S \cup B} m_j \int_0^{\bar{s}^j_i} x \frac{\delta^j_i}{\delta^j_i} dx = \sum_{j \in A \cup S \cup B} m_j \int_0^{\bar{s}'^j_i} x \frac{\delta^j_i}{\delta^j_i} dx \]
but \( \lambda_i \neq \lambda_{-i} \). Q.E.D.

Let \( \lambda \equiv \lambda_i = \lambda_{-i} \). The following lemma establishes the value of \( \bar{s}^j_i \).

**Lemma 5:** \( \bar{s}^j_i = \frac{\delta^j_i}{\lambda} \) \( \forall \ i \) and \( j \).

**Proof:** From lemmas 3 and 4, we know that for each party \( i \) and any segment \( j \)
\[ \frac{1}{\lambda} F^{-j}_i \left( \frac{x \delta^j_i}{\delta^j_i} \right) - x \]
is constant \( \forall x \in (0, \bar{s}^j_i) \). It then follows that party \( i \) would never use a strategy that provides offers in \( \left( \frac{1}{\lambda}, \infty \right) \) since an offer of zero strictly dominates such a strategy. The result follows directly. Q.E.D.
The following lemma establishes that there exists a unique $\lambda$ that satisfies the budget constraint.

**Lemma 6:** There exists a unique value for $\lambda$, and this value is

$$1 - \frac{1}{2} \sum_{j \in A} m_j (1 - \delta^j_A) - \frac{1}{2} \sum_{k \in B} m_k (1 - \delta^k_B) = \frac{1 - \sigma_A - \sigma_B}{2}.$$

**Proof:** The budget constraint determines the unique value of $\lambda$. Thus, $\lambda$ solves

$$\sum_{j \in A \cup S \cup B} m_j \int_0^{\delta^j_i} x \frac{\delta^j_i}{\delta^j_i} dx = 1$$

Solving for $\lambda$ we have that

$$\lambda = \frac{1 + \sum_{j \in A} m_j (\delta^j_A - 1) + \sum_{k \in B} m_k (\delta^k_B - 1)}{2} = \frac{1 - \sigma_A - \sigma_B}{2}$$

Q.E.D.

This completes the proof of Theorem 2.

To construct the unique Nash equilibrium of the redistributive politics game, note that the intensity of attachment parameters, $a^j_i = 1 - \delta^j_i$, are isomorphic to differences in bidders’ valuations in an all-pay auction. Thus, in each segment of voters loyal to party $-i$, party $i$ places mass equal to $\frac{1 - \delta_{-i}}{\lambda} = 1 - \delta_{-i}$ at 0. Then letting $z \equiv \frac{1}{\lambda}$, the unique equilibrium is for party $A$ to offer redistribution according to

$$\forall j \in A \quad F^j_A (x) = \frac{x}{z \delta^j_A} \quad x \in [0, z \delta^j_A]$$

$$\forall k \in B \quad F^k_A (x) = \frac{x}{z} \quad x \in [0, z]$$

$$F^S_A (x) = \frac{x}{z} \quad x \in [0, z]$$
and for party $B$ to offer redistribution according to

\[
\forall k \in B \quad F^k_B(x) = \frac{x}{z\delta^k_B} \quad x \in [0, z\delta^k_B]
\]

\[
F^S_B(x) = \frac{x}{z} \quad x \in [0, z]
\]

\[
\forall j \in A \quad F^j_B(x) = (1 - \delta^j_A) + \frac{\delta^j_A x}{z} \quad x \in [0, z]
\]

where $z = \frac{1}{\lambda} = \frac{2}{1 - \sigma_A - \sigma_B}$. 
Figure 1(a): Transfers from Party $i$’s Equilibrium Redistribution Schedule

Figure 1(b): Utilities from Party $i$’s Equilibrium Redistribution Schedule
Figure 2: Level Curves of Party Polarization