Chapter 6: Valuing Bonds

Note: Read the chapter then look at the following.

Fundamental question: What are the cash flows associated with bonds?

6.1 Bond Cash Flows, Prices and Yields

A. Bond Terminology

Terms: bond certificate, maturity date, term, coupons, face value, coupon rate

$$CPN = \frac{CR \times FV}{CPY} \tag{6.1}$$

where:

CPN = coupon payment
CR = coupon rate
FV = face value of bond
CPY = number of coupon payments per year

Ex. Assume a bond with a $1000 face value pays a 10% coupon rate. What coupon does the issuer promise to pay bondholders if the coupons are paid semiannually (as most are)?

$$CPN = \frac{.1 \times 1000}{2} = 50$$

Video Solution

B. Zero-Coupon Bonds

=> bonds that pay no cash flows other than a maturity value

Terms: Treasury bills, discount, pure discount bonds, spot interest rates, zero-coupon yield curve

1. Yield to Maturity

Notes:

1) Yield to maturity = special name of internal rate of return (IRR) on a bond
   => discount rate that sets present value of promised bond payments equal to current market price of bond
2) If bond is risk-free, yield to maturity is the same as the IRR from chapter 4.
3) Do not really need the following equations. Can use equation (4.2) to solve for present value (to get price) or to solve for "r" to get YTM.

$$P = \frac{FV}{(1+YTM_n)^n} \tag{6.2}$$

$$YTM_n = \left(\frac{FV}{P}\right)^{1/n} - 1 \tag{6.3}$$

where:

YTM_n = yield to maturity from holding the bond from today until matures on date n

Ex. Assume a zero-coupon bond pays $1000 when it matures 5 years from today and that the yield to maturity on the bond equals 4.5%. What is the price of the bond?

Timeline

$$P = \frac{1000}{(1.045)^5} = 802.45$$

Video Solution

Ex. Assume the price of the previous bond rises to $810. What is the yield to maturity on the bond?

Timeline

$$YTM_5 = \left(\frac{1000}{810}\right)^{1/5} - 1 = .04304$$

Video Solution

2. Risk-free Interest Rates

=> the risk-free interest rate for a maturity of n years equals the yield to maturity on a risk-free bond that matures n years from today.

$$r_n = YTM_n \tag{6.4}$$

C. Coupon Bonds

=> Coupon bonds pay par at maturity. They also pay a coupon at maturity and pay a coupon every period (usually semiannually) before this.

Note: We will assume semiannual coupons unless told otherwise.

Terms: Treasury notes, Treasury bonds

Note: Do not really need the following equation. Can combine equations (4.2) and (4.9) or (4.12, if assume g = 0).

$$P = CPN \times \frac{1}{y}\left(1 - \left(\frac{1}{(1+y)^N}\right)\right) + \frac{FV}{(1+y)^N} \qquad (6.5)$$

where:

y = return per coupon period on coupon bond

Notes:

1) Footnote #3 on p. 173 is important. You need to understand how to value bonds at all dates…not just at coupon dates.
2) YTM (an APR) = y x N
Can calculate effective annual rate from rate per coupon interval. But the rate normally quoted for bonds is the APR. To compare the returns on bonds with different coupon intervals, need to compare effective annual interest rates (APYs).

Ex. Assume a bond matures for $1000 six years from today and has a 7% coupon rate with semiannual coupons. What is the value of the bond today if the yield to maturity on the bond equals 8.5%?

Timeline

$CPN = \frac{.07 \times 1000}{2} = 35$

$y = \frac{.085}{2} = .0425$

$P = \frac{35}{.0425}\left(1 - \left(\frac{1}{1.0425}\right)^{12}\right) + \frac{1000}{(1.0425)^{12}} = 930.62$

Video Solution

Ex. Assume a bond matures for $1000 seven years from today and had a 9.5% coupon rate. What is the yield to maturity on the bond if the price today is $1050?

Timeline

$$CPN = \frac{.095 \times 1000}{2} = 47.5$$

$$P = \frac{47.5}{y}\left(1 - \left(\frac{1}{1+y}\right)^{14}\right) + \frac{1000}{(1+y)^{14}} = 1050$$

Using goal seek in Excel:

y = .04268
=> YTM = yield to maturity = .0854 = .04268 x 2

Video Solution

Concept Check: All

6.2 Dynamic Behavior of Bond Prices

A. Discounts and Premiums

Terms: premium, par
Key issues:

1) coupon rate vs. yield to maturity
2) return on bond driven by coupons and change in price
3) over time, bond prices tend to move towards par value
4) bond prices deviate from this trend because of two reasons
   => fall on coupon payments, rise between coupon payments
   => rise if interest rate falls and fall if interest rate rises
      Reason: present value of future cash flows rise when interest rates fall and fall when interest rates rise

B. Time and Bond Prices

Key issues:

1) bond prices must eventually end up at par (+ coupon) just before maturity
   => generally drives price from current price towards par
   => see Figure 6.1 in textbook
2) if interest rates don't change, will earn yield to maturity over time hold bond

C. Interest Rate Changes and Bond Prices

Key issue: sensitivity of bond price to changes in interest rate depends on bond's duration

Note: Duration is basically the average maturity of the bond's cash flows (coupons and par value). So the longer the duration, the longer the average maturity of the bond's cash flow…and the more sensitive the bond to changes in interest rates.

Ex. Assume the interest rate for all maturities is 5%.

Q1: What is the price today of a 5-year bond paying a 10% coupon rate with annual coupons?

Q2: What is the price today of a 30-year zero coupon bond?

Q3: Which bond's price will have a larger percentage change if interest rates rise to 8%?

Note: Duration of 5-year bond is less than 5 years since some cash flows before maturity. Duration of 30-year bond is 30 years since only one cash flow.

Q1: $P = \frac{100}{.05}\left(1 - \left(\frac{1}{1.05}\right)^5\right) + \frac{1000}{(1.05)^5} = 1216.47$

Q2: $P = \frac{1000}{(1.05)^{30}} = 231.38$

Q3:

Coupon bond price falls by 11% [(1079.85-1216.47)/1216.47] to $1079.85

$$P = \frac{100}{.08}\left(1 - \left(\frac{1}{1.08}\right)^5\right) + \frac{1000}{(1.08)^5} = 1079.85$$

Timeline
Video Solution

Zero-coupon bond price falls 57% [(99.38-231.38)/231.38] to $99.38

$$P = \frac{1000}{(1.08)^{30}} = 99.38$$

Timeline
Video Solution

Note: Price of a 5-year zero coupon bond falls 13% from 783.53 to 680.58

Note: duration of the zero-coupon bond is 5 years rather than less than 5 years.

1. Clean and Dirty Prices for Coupon Bonds

$$CP = DP - CPN\left(\frac{DSLC}{DICP}\right) \tag{6.A}$$

where:

CP = clean price
DP = dirty price = cash price
CPN = coupon
DSLC = days since last coupon
DICP = days in coupon period

Note: can also use MSLC (months since last coupon) and MICP (months in coupon period)

Note: Accrued interest is linear while the change in a bond's price over time is not (due to compounding). Thus the clean price will still have a very slight saw-tooth pattern over time if interest rates do not change.

Ex. Assume that a bond with an 8.5% coupon rate (semiannual coupons) matures ten years from today.

a. What is the value (and price) of the bond today (per $100 of face value) if the yield to maturity equals 5%?

b. Assume that four months have elapsed (maturity is now 9 years and 8 months from today), but the yield to maturity on the bond has not changed. What is the value, clean price, and dirty price of the bond?

c. Assume that the bond matures 9 years and 8 months from today (four months have elapsed), but that the yield to maturity has fallen to 4% instead of remaining at 5%. What is the value, clean price, and dirty price of the bond?

d. Assume that the bond matures 9 years and 8 months from today and that the clean price of the bond is $120. What is the yield to maturity on the bond?

Coupon = $\frac{.085 \times 100}{2} = 4.25$

y = effective rate per coupon period = $\frac{.05}{2} = .025$

a. $V_0 = \frac{4.25}{.025}\left(1 - \left(\frac{1}{1.025}\right)^{20}\right) + \frac{100}{(1.025)^{20}} = 127.2810$

Timeline

Video Solution

b.

$$V_{-4mo} = \frac{4.25}{.025}\left(1 - \left(\frac{1}{1.025}\right)^{20}\right) + \frac{100}{(1.025)^{20}} = 127.2810$$

Note: No coupons have been paid and interest rates have not changed

=> the value one coupon-period before first coupon is unchanged.

$$V_0 = 127.2810(1.025)^{4/6} = 129.3936$$

Dirty price = 129.3936

Clean price = $129.3936 - 4.25\left(\frac{4}{6}\right) = 126.5603$

Note: A more precise answer would use days rather than months. But in this case, we would really need to calculate the value of each coupon payment and the par value separately based on how many days from today each payment is made.

Timeline

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c.

$V_0 = \left(\frac{4.25}{.02}\left(1 - \left(\frac{1}{1.02}\right)^{20}\right) + \frac{100}{(1.02)^{20}}\right)(1.02)^{4/6} = 138.6086$ = dirty price

Clean price = $138.6086 - 4.25\left(\frac{4}{6}\right) = 135.7752$

Video Solution

d. Cash price = $120 + 4.25\left(\frac{4}{6}\right) = 122.8333$

$\left(\frac{4.25}{y}\left(1 - \left(\frac{1}{1+y}\right)^{20}\right) + \frac{100}{(1+y)^{20}}\right)(1 + y)^{4/6} = 122.8333$

=> solving for y (using solver in Excel): y = .02885

=> yield to maturity (YTM) = .0577 = .02885 × 2

Timeline

Concept Check: All