Chapter 15: Debt and Taxes

Fundamental Question: How do taxes affect capital structure decisions?

15.1 The Interest Tax Deduction

\[ ITS = \tau_c \times IP \quad (15.1) \]

where:
\( ITS \) = interest tax shield
\( \tau_c \) = corporate tax rate
\( IP \) = interest paid

15.2 Valuing the Interest Tax Shield

\text{\( \Rightarrow \) calculate present value of annual tax saving.}

A. The Interest Tax Shield and Firm Value

Ex. Assume you are starting a firm that will generate a pre-tax cash flow of $100,000 per year. What are your after-tax cash flows if you call your investment equity and if you call your investment a loan? Assume the corporate tax rate is 35% and that your firm pays out all of its cash flows either as dividends or interest.

If paid as a dividend: \( 100,000 - .35(100,000) = $65,000 \)
If paid as interest: $100,000

Video Solution

\( \Rightarrow \) extra $35,000 per year if call investment a loan

Q: Where does the $35,000/year come from?
Q: If we only consider corporate taxes, how should firms be funded?

\[ CFL = CF^U + ITS \quad (15.A) \]

where:
\( CFL \) = cash flows to investors with leverage
\( CF^U \) = cash flows to investors without leverage

Note: the increased cash flow comes at the expense of the government which receives less taxes

Ex.
\[ CF^U = 100,000(1 - 0.35) = 65,000 \]
\[ CFL = 65,000 + .35(100,000) = 100,000 \]

Video Solution
MM Proposition I with Corporate Taxes: The total value of the levered firm exceeds the value of the firm without leverage due to the present value of the tax savings from debt.

Let:

\[ V^L = \text{value of levered firm (firm with debt and equity financing)} \]
\[ V^U = \text{value of unlevered firm (firm funded only with equity)} \]

\[ V^L = V^U + PV(ITS) \]  
(15.2)

B. The Interest Tax Shield with Permanent Debt

Variables that can change and thus affect amount of firm’s tax shield over time: 1) amount of debt outstanding, 2) interest rates, 3) failure to make interest payments, 4) tax code, 5) firm’s income tax bracket, 6) firm maintains certain debt ratio but grows (or shrinks)

Notes:

1) permanent debt is special case to demonstrate impact of taxes on capital structure decisions  
2) most corporate debt is essentially “permanent” since refinance rather than paying off

\[ D = PV(IP) \]  
(15.3)

where:

\[ D = \text{debt} \]
\[ IP = \text{interest payments} \]

\[ PV(ITS) = \tau_c \times D \]  
(15.4)

where:

\[ \tau_c = \text{corporate tax rate} \]

Reason:

\[ PV(ITS) = \frac{\tau_c \times IP}{r} \] : if debt is permanent, interest tax shield is a perpetuity
\[ = \frac{\tau_c \times r \times D}{r} \] : IP (interest payments) = interest rate x debt
\[ = \tau_c \times D \] : as long as tax rate fixed, interest tax shield and interest payments have same risk. As a result, \( r \) cancels out.
Value of firm with permanent debt:

$$V^L = V^U + \tau_c D$$ \hspace{1cm} (15.5)

Ex. Assume firm is currently funded with 100% equity and has a market value of $500,000. What will the firm be worth if the corporate tax rate is 35%, and the firm issues $200,000 of permanent debt and uses the proceeds to repurchase stock?

$$V^L = 500,000 + .35(200,000) = 570,000$$

=> U.S. corporate tax code gives firms an incentive to issue debt

Q: Where does the extra $70,000 come from?

Note: if corporate tax rate equals 40%, every dollar of permanent debt raises the value of the firm by $0.40.

C. The Weighted Average Cost of Capital with Taxes

$$r_{wacc} = \left( \frac{E}{E+D} \right) r_E + \left( \frac{D}{E+D} \right) r_d (1 - \tau_c)$$ \hspace{1cm} (15.6)

where:

- $E$ = market value of firm’s outstanding equity
- $D$ = market value of firm’s outstanding debt
- $r_E$ = cost of capital for levered equity
- $r_D$ = cost of capital for firm’s debt

Note: this equation is the same as equation (12.12)

Note: this equation is essentially the same as (12.13)

Ex. Assume that the market value of a firm’s equity is $300,000 and that the market value of its debt is $200,000. Assume also that the cost of equity is 12%, that the cost of debt is 5%, and that the corporate tax rate is 35%. What is the cost of capital for the firm if interest is not tax deductible and if it is?

$$r_U = \left( \frac{300,000}{500,000} \right) .12 + \left( \frac{200,000}{500,000} \right) .05 = .092$$

$$r_{wacc} = \left( \frac{300,000}{500,000} \right) .12 + \left( \frac{200,000}{500,000} \right) .05(1 - .35) = .085$$

$$= .092 - \left( \frac{200,000}{500,000} \right) (.35)(.05)$$
Q: What happens to the value of the firm today if the cost of capital is lower?

   Increases

D. The Interest Tax Shield with a Target Debt-Equity Ratio

Concept Check: all

15.3 Recapitalizing to Capture the Tax Shield

A. The Tax Benefit

B. The Share Repurchase

C. No Arbitrage Pricing

   Key issue: When securities fairly priced, the original shareholders of the firm capture the full benefit of the interest tax shield from an increase in leverage.

   => existing shareholders will not sell back their shares (as part of the recapitalization) unless the price reflects the benefit of the tax shield

D. Analyzing the Recap: The Market Value Balance Sheet

Concept Check: all

15.4 Personal Taxes

A. Including Personal Taxes in the Interest Tax Shield

   Keys:

   1) value of firm = value of securities issued by firm
   2) value of securities to investors depends on their after-tax cash flows

   => value of firm depends on personal as well as corporate taxes

   3) interest income is typically taxed at a higher rate than equity income

   => personal taxes reduce (and might even eliminate) the corporate tax benefit of debt
\( \tau^* = 1 - \frac{(1-\tau_c)(1-\tau_e)}{(1-\tau_i)} \) \hspace{1cm} (15.7)

where:

\( \tau^* = \text{effective tax advantage of debt} \)
\( \tau_c = \text{corporate tax rate} \)
\( \tau_e = \text{personal tax rate on equity income} \)
\( \tau_i = \text{personal tax rate on interest income} \)

Note: if want to see where this equation comes from, see proof in section 15.A below

B. Valuing the Interest Tax Shield with Personal Taxes

\[ V^L = V^U + \tau^* D \] \hspace{1cm} (15.8)

Notes:

1) as long as \( \tau^* > 0 \), value of firm rises if issues additional debt (and repurchases shares with proceeds)
2) personal taxes affect the firm’s WACC indirectly as the cost of capital of debt and equity rise to compensate investors for their taxes

Ex. Assume that \( \tau_c = .35, \tau_i = .3, \) and \( \tau_e = .3 \)

\[ \tau^* = 1 - \frac{(1-.35)(1-.3)}{1-.3} = .35 \]

\( \Rightarrow V^L = V^U + .35D \)

Note: if \( \tau_e = \tau_i \): personal taxes cancel out

Q: How should we fund the firm?

Ex. Assume that \( \tau_c = .35, \tau_i = .3, \) and \( \tau_e = .15 \)

\[ \tau^* = 1 - \frac{(1-.35)(1-.15)}{1-.3} = .2107 \]

\( \Rightarrow V^L = V^U + .2107D \)

Q: How should we fund the firm?

Note: if \( \tau_i > \tau_e \): additional taxes at personal level reduces tax benefit at corporate level
C. Determining the Actual Tax Advantage of Debt

Note: Difficult to calculate $\tau^*$ for several reasons

1. investors don’t have to realize capital gains and can offset gains with losses
2. mix of dividends and capital gains varies by firm
3. returns on retirement accounts untaxed
4. tax rates vary across investors

Concept Check: all

15.5 Optimal Capital Structure with Taxes

A. Do Firms Prefer Debt?

B. Limits to the Tax Benefit of Debt

Key => interest is only tax deductible for firm if has enough EBIT (earnings before interest and taxes) to deduct the interest

=> if can’t deduct taxes, corporate tax savings from interest paid is zero

=> can set $\tau_c$ to 0 in equation (15.7)

Note: For simplicity we will ignore carry backs and carry forwards. With carry backs, firms can get back taxes paid in previous two years (by using current losses to offset pervious profits). With carry forwards, firms can avoid taxes on future profits by using current losses to offset future profits.

$$\tau^*_{ex} = 1 - \frac{(1-0)(1-\tau_c)}{(1-\tau_i)} = \frac{\tau_e - \tau_i}{1-\tau_i}$$

where: $\tau^*_{ex} =$ effective tax advantage on interest in excess of EBIT

Note: usually, $\tau^*_{ex} < 0$ since normally $\tau_e < \tau_i$

C. Optimal Leverage with Risk-free Earnings Before Interest and Taxes (EBIT)

General idea: issue debt as long as $\tau^*$ positive
Example: Assume that a firm’s riskless EBIT will be $25,000 per year forever. Assume also that the corporate tax rate is 35%. Finally, assume that the tax rate for individual interest income is 30% and that the tax rate on individual equity income is 20%.

Calculate the effective tax advantage of debt if interest < $25,000 per year and if it is ≥ $25,000 per year.

=> if interest < $25,000 per year, \( \tau^* = 1 - \frac{(1-0.35)(1-0.2)}{(1-0.3)} = 0.25714 \)

=> if interest ≥ $25,000 per year, \( \tau^* = 1 - \frac{(1-0)(1-0.2)}{(1-0.3)} = -0.1429 = \frac{0.2-3}{1-3} = \tau^* \)

**Video Solution**

Q: What do these numbers tell us?

Optimal debt: set interest equal to EBIT

=> no corporate taxes paid

=> optimal annual interest in example = $25,000

D. Optimal Leverage with Risky Earnings (EBIT)

=> as interest expense increases, chance of deducting the interest falls

\[ E(\tau_C) = \text{expected corporate tax savings from interest} \]

= probability of deducting additional interest x \( \tau_C \)
\( \tau^* = 1 - \frac{(1-E(\tau_c))(1-\tau_e)}{(1-\tau_d)} \)  

(15.7B)

Key: issue debt as long as \( \tau^* \) positive

Ex. Assume that the corporate tax rate is 35\%, that the personal tax rate is 30\% on interest income and 20\% on equity income. Assume also the following probability distribution for the firm’s EBIT.

<table>
<thead>
<tr>
<th>EBIT</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>.15</td>
</tr>
<tr>
<td>25,000</td>
<td>.30</td>
</tr>
<tr>
<td>35,000</td>
<td>.35</td>
</tr>
<tr>
<td>45,000</td>
<td>.20</td>
</tr>
</tbody>
</table>

Q: What is the probability of deducting interest if:

1) between 0 and 10,000? 1 (100\%)
   => firm assured of deducting since lowest possible EBIT is 10,000

2) between 10,000 and 25,000? 0.85 (85\%)
   => firm can deduct interest if EBIT equals 25,000 or 35,000 or 45,000
   Q: What is the probability of having EBIT of 25,000, or 35,000 or 45,000?
   
   \(.85 = .3 + .35 + .2\)

3) between 25,000 and 35,000? 0.55 (55\%)
   => firm can deduct interest if EBIT equals 35,000 or 45,000
   Q: What is the probability of having EBIT of 35,000 or 45,000?
   
   \(.55 = .35 + .2\)

4) between 35,000 and 45,000? .2 (20\%)
   => firm can deduct interest if EBIT = 45,000
   Q: What is the probability of having EBIT of 45,000?
   
   \(.2\)

5) more than 45,000? 0\%
   => firm can’t deduct since highest possible EBIT is 45,000.

[Video Solution]
Q: What is $E(\tau_C)$ and $\tau^*$ for interest paid between:

1) $0$ and $10,000$?
   Q: Issue debt if interest currently less than $10,000$?

2) $10,000$ and $25,000$?
   Q: Issue debt if interest currently between $10,000$ and $25,000$?

3) $25,000$ and $35,000$?
   Q: Issue debt if interest currently between $25,000$ and $35,000$?

4) $35,000$ and $45,000$?
   Q: Issue debt if interest currently between $35,000$ and $45,000$?

5) more than $45,001$?
   Q: Issue debt if interest exceeds $45,000$?
1) $0 and $10,000?

=> E(τC) = .35 = 1 (.35)
note: multiplying probability firm can deduct interest by τc

=> τ* = 1 - \frac{(1-.35)(1-.2)}{(1-.3)} = .25714

Q: Issue debt if interest currently less than $10,000? yes

2) $10,000 and $25,000?

=> E(τC) = .85(.35) = .2975
note: multiplying probability firm can deduct interest by τc

=> τ* = 1 - \frac{(1-.2975)(1-.2)}{(1-.3)} = .19714

Q: Issue debt if interest currently between $10,000 and $25,000? yes

3) $25,000 and $35,000?

=> E(τC) = .55(.35) = .1925

=> τ* = 1 - \frac{(1-.1925)(1-.2)}{(1-.3)} = .07714

Q: Issue debt if interest currently between $25,000 and $35,000? yes

4) $35,000 and $45,000?

=> E(τC) = .2(.35) = .07

=> τ* = 1 - \frac{(1-.07)(1-.2)}{(1-.3)} = - .06286

Q: Issue debt if interest currently between $35,000 and $45,000? no

5) more than $45,001? E(τC) = 0; τ* = -.14286 = τ_{exit}

Q: Issue debt if interest exceeds $45,000? no

Video Solution

=> optimal annual interest = $35,000
E. Growth and Debt

1. A firm’s optimal debt is related to ability to deduct interest which depends on current earnings.
2. A firm’s equity values reflect both current and future earnings and cash flows.

=> Higher growth firms will tend to have higher equity values other things equal and as a result lower debt ratios.

Result: the higher the firm’s growth, the lower its optimal proportion of debt to value \([D/(D+E)]\).

F. Other Tax Shields

G. The Low Leverage Puzzle
15.A Proof of Equations (15.7) and (15.8)

Note: You are not required to know the following, but I provide it in case you want to know the logic behind the equations for the effective tax advantage of debt (15.7) and the value of the firm including interest tax shield once include personal taxes (15.8).

Let:

\[ CF_A = \text{cash flow generated by firm’s assets each year} \]
\[ \tau_c = \text{tax rate on corporate income} \]
\[ \tau_i = \text{tax rate on interest income for individuals} \]
\[ \tau_e = \text{tax rate on equity income for individuals} \]
\[ D = \text{permanent debt level} \]
\[ r_U = \text{after-tax return demanded by stockholders of unlevered firm} \]
\[ r_D = \text{interest rate paid on debt} \]

Note: the after-tax return for bondholders = \( r_D(1-\tau_i) \)
=> bondholders will discount the after-tax payments they receive at \( r_D(1-\tau_i) \)

If firm has no debt:

\[ \text{CF to stockholders} = CF_A (1-\tau_c)(1-\tau_e) \] (1)

\[ V^U = \frac{CF_A (1-\tau_c)(1-\tau_e)}{r_u} \] (2)

If firm has debt:

\[ \text{CF to stockholders} = (CF_A - r_D D)(1-\tau_c)(1-\tau_e) \] (3)

\[ \text{CF to debt holders} = r_D D \] (4)

=> CF to investors = \( (CF_A - r_D D)(1-\tau_c)(1-\tau_e) + r_D D(1-\tau_i) \) (5)

\[ = CF_A(1-\tau_c)(1-\tau_e) - r_D D(1-\tau_c)(1-\tau_e) + r_D D(1-\tau_c)(1-\tau_e)(1-\tau_i) \] (6)

\[ = CF_A(1-\tau_c)(1-\tau_e) + r_D D((1-\tau_c)(1-\tau_e) - (1-\tau_i)) \] (7)

\[ V^I = \frac{CF_A(1-\tau_c)(1-\tau_e) + r_D D((1-\tau_i)(1-\tau_c))}{r_u} \] (8)

Notes:

1) equivalent to take the present value of an entire cash flow or to take the present value of the parts and add up the present values.

2) since first piece of cash flow in (7) is same as the cash flow to the stockholders of an unlevered firm (1), should use same rate as for an unlevered firm (\( r_U \))
3) since the second piece of the cash flow in (7) is a fixed percent of the interest payments \((r_D D)\), you can use the same rate on this second piece as you can on the interest payments themselves…and the appropriate after-tax discount rate is thus \(r_D (1 - \tau)\)

Substituting equation (2) for the first part of (8) and canceling out \(r_D\) we get:

\[
V^L = V^U + \frac{D \left( (1 - \tau_i) - (1 - \tau_e)(1 - \tau_e) \right)}{(1 - \tau_i)} = V^U + \left( \frac{(1 - \tau_i) - (1 - \tau_e)(1 - \tau_e)}{(1 - \tau_i)} \right) D
\]  

\[
\Rightarrow V^L = V^U + \left( 1 - \frac{(1 - \tau_e)(1 - \tau_e)}{(1 - \tau_i)} \right) D
\]  

\[
\Rightarrow V^L = V^U + \tau^* D
\]

where: \(\tau^* = 1 - \frac{(1 - \tau_e)(1 - \tau_e)}{(1 - \tau_i)}\)