Chapter 14: Supplement

Leverage, Risk, and Return

Let:

E = market value of the firm's outstanding equity
D = market value of the firm's outstanding debt
β_E = beta of firm's levered equity
β_D = beta of firm's debt
β_U = beta of firm's unlevered equity (if it has no debt)

1. Leverage, risk, and the cost of equity capital

1) Unlevered equity is equivalent to a portfolio of debt and levered equity

2) The beta of a portfolio is a weighted average of the portfolio's betas

$$\Rightarrow \beta_U = \left(\frac{E}{E+D}\right)\beta_E + \left(\frac{D}{E+D}\right)\beta_D \qquad (14.9)$$

=> solving for E: $\beta_E = \beta_U + \frac{D}{E}(\beta_U - \beta_D)$ (14.10)

=> since $\beta_D < \beta_U$, β_E increases as increase leverage (increase debt and reduce equity)

3) Impact of leverage on equity cost of capital:

$$r_E = r_f + \beta_E\left(E(R_{Mkt}) - r_f\right) \qquad (12.4)$$

$$r_E = r_f + \left(\beta_U + \frac{D}{E}(\beta_U - \beta_D)\right)\left(E(R_{Mkt}) - r_f\right)$$

Note: substitute 14.10

$$r_E = r_f + \beta_U\left(E(R_{Mkt}) - r_f\right) + \frac{D}{E}\left(\beta_U\left(E(R_{Mkt}) - r_f\right) - \beta_D\left(E(R_{Mkt}) - r_f\right)\right)$$

Note: distributive property

$$r_E = r_f + \beta_U\left(E(R_{Mkt}) - r_f\right) + \frac{D}{E}\left(\left(r_f + \beta_U\left(E(R_{Mkt}) - r_f\right)\right) - \left(r_f + \beta_D\left(E(R_{Mkt}) - r_f\right)\right)\right)$$

Note: adding and subtracting r_f

$$r_E = r_U + \frac{D}{E}(r_U - r_D) \qquad (14.5)$$

=> since $r_D < r_U$, r_E increases as increase leverage

2. Leverage and expected return

$$E(R_U) = \left(\frac{E}{E+D}\right)E(R_E) + \left(\frac{D}{E+D}\right)E(R_D)$$

Note: if own all of a firm's equity and debt, have created an unlevered position in the firm

$$E(R_E) = E(R_U) + \left(\frac{D}{E}\right)(E(R_U) - E(R_D))$$

Note: solving for $E(R_E)$

3. Weighted-Average Cost of Capital and Leverage

$$r_{WACC} = \left(\frac{E}{D+E}\right) r_E + \left(\frac{D}{D+E}\right) r_D$$

$$= \left(\frac{E}{D+E}\right)\left(r_U + \frac{D}{E}(r_U - r_D)\right) + \left(\frac{D}{D+E}\right) r_D$$

Note: substituting 14.5

$$= \left(\frac{E}{D+E}\right) r_U + \left(\frac{E}{D+E}\right)\frac{D}{E}(r_U - r_D) + \left(\frac{D}{D+E}\right) r_D$$

Note: distributive property

$$= \left(\frac{E}{D+E}\right) r_U + \left(\frac{D}{D+E}\right)(r_U - r_D) + \left(\frac{D}{D+E}\right) r_D$$

Note: $\left(\frac{E}{D+E}\right)\frac{D}{E} = \left(\frac{D}{D+E}\right)$

$$= \left(\frac{E}{D+E}\right) r_U + \left(\frac{D}{D+E}\right) r_U - \left(\frac{D}{D+E}\right) r_D + \left(\frac{D}{D+E}\right) r_D$$

Note: distributive property

$$= \left(\frac{D+E}{D+E}\right) r_U = r_U$$

Notes:

1) $-\left(\frac{D}{D+E}\right) r_D + \left(\frac{D}{D+E}\right) r_D = 0$

2) $\left(\frac{E}{D+E}\right) r_U + \left(\frac{D}{D+E}\right) r_U = \left(\frac{E+D}{D+E}\right) r_U = \left(\frac{D+E}{D+E}\right) r_U$

3) $\left(\frac{D+E}{D+E}\right) = 1$