Chapter 10: Capital Markets and the Pricing of Risk

Big Picture:

1) To value a project, we need an interest rate to calculate present values
2) The interest rate will depend on the risk of the project
3) To determine this interest rate, we need to be able to:
   a) measure returns
   b) measure risk
   c) figure out the relationship between risk and return

I. Estimating Risk and Return

Basic approaches:

1) make forecasts about the future
2) look at the past and assume future will be like the past.

A. Estimates based on forecasts

Key: need probability distributions for investments
    => probability \( p_R \) of each possible return \( R \)

1. Expected return: \( E[R] = \sum R p_R \times R \) \hspace{1cm} (10.1)

    => return would earn on average if invest in assets over and over and if the distribution does not change

2. Variance \( \text{Var}(R) \) and standard deviation \( \text{SD}(R) \):

    \[
    \text{Var}(R) = \sum R p_R \times (R - E(R))^2 \hspace{1cm} (10.2)
    \]
    \[
    \text{SD}(R) = \sqrt{\text{Var}(R)} \hspace{1cm} (10.3)
    \]

    => measure of how widely scattered the possible returns are
    => the higher the number, the more widely scattered the possible returns
Chapter 10: Capital Markets and the Pricing of Risk

Ex. Given the following possible returns on General Electric (GE) stock, what is the expected return and standard deviation of returns on GE stock next year?

<table>
<thead>
<tr>
<th>Economy</th>
<th>Probability</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bust</td>
<td>.25</td>
<td>-26%</td>
</tr>
<tr>
<td>Normal</td>
<td>.40</td>
<td>11%</td>
</tr>
<tr>
<td>Boom</td>
<td>.35</td>
<td>44%</td>
</tr>
</tbody>
</table>

\[
E[R] = .25(-26) + .4(11) + .35(44) = 13.3\%
\]

\[
\text{Var}(R) = .25(-26-13.3)^2 + .4(11-13.3)^2 + .35(44-13.3)^2 = 718.11
\]

\[
\text{SD}(R) = \sqrt{718.11} = 26.8\%
\]

Ex. How does an investment in GE compare to an investment in General Mills (GIS) if the expected return on GIS is 5% and that the standard deviation of returns on GIS is 10%?

=> General Mills has a lower expected return but less volatility than GE.

Q: Is it better to own GE or General Mills?

Note: if returns were normally distributed, then can compare the distributions of GE and GIS.

Note: GE’s distribution is more spread out and centered further to the right.

=> greater risk and higher expected return
B. Estimates based on historical returns

Key assumption: future will be like the past

1. Realized return: \( R_{t+1} = \frac{D_{t+1}}{P_t} + \frac{P_{t+1} - P_t}{P_t} \)  

\[(10.4)\]

Notes:

1) \( R_{t+1} \) = return actually earned between \( t \) and \( t+1 \)
2) \( D_{t+1} \) = dividend at \( t+1 \)
3) \( P_t \) = stock price at \( t \)
4) \( P_{t+1} \) = stock price at \( t+1 \)
5) \( \frac{D_{t+1}}{P_t} \) = dividend yield
6) \( \frac{P_{t+1} - P_t}{P_t} \) = capital gains yield
7) must calculate a return any time a dividend is paid
8) can calculate at any non-dividend date by assuming a dividend of 0

Ex. Assume the following prices and dividends for General Electric (GE) stock

<table>
<thead>
<tr>
<th>Date</th>
<th>Dividend</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31</td>
<td>$0.00</td>
<td>$15.13</td>
</tr>
<tr>
<td>2/25</td>
<td>$0.10</td>
<td>$15.92</td>
</tr>
<tr>
<td>6/17</td>
<td>$0.10</td>
<td>$15.91</td>
</tr>
<tr>
<td>9/16</td>
<td>$0.12</td>
<td>$16.23</td>
</tr>
<tr>
<td>12/22</td>
<td>$0.14</td>
<td>$18.06</td>
</tr>
<tr>
<td>12/31</td>
<td>$0.00</td>
<td>$18.29</td>
</tr>
</tbody>
</table>

What was return between 9/16 and 12/22?

\[ R_{12/22} = .1214 = 12.14\% \]

\[ = \frac{.14}{16.23} + \frac{18.06 - 16.23}{16.23} = .0086 + .1128 \]

Q: What does this tell us about GE?

2. Realized return over longer periods

Key: usually think in terms of annual returns

a. Allow dividend-period returns to compound

\[ 1 + R_L = (1 + R_{S1})(1 + R_{S2})(1 + R_{S3}) \ldots \]

\[(10.5)\]

Note: assumes reinvesting all of dividends so earn return on them
Ex. Returns per period (previous GE example):

<table>
<thead>
<tr>
<th>Date</th>
<th>Dividend</th>
<th>Price</th>
<th>Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31</td>
<td>$0.00</td>
<td>$15.13</td>
<td>n.a.</td>
</tr>
<tr>
<td>2/25</td>
<td>$0.10</td>
<td>$15.92</td>
<td>5.88%</td>
</tr>
<tr>
<td>6/17</td>
<td>$0.10</td>
<td>$15.91</td>
<td>0.57%</td>
</tr>
<tr>
<td>9/16</td>
<td>$0.12</td>
<td>$16.23</td>
<td>2.77%</td>
</tr>
<tr>
<td>12/22</td>
<td>$0.14</td>
<td>$18.06</td>
<td>12.14%</td>
</tr>
<tr>
<td>12/31</td>
<td>$0.00</td>
<td>$18.29</td>
<td>1.27%</td>
</tr>
</tbody>
</table>

\[1 + R_{\text{year}} = 1.2427 = (1.0588)(1.0057)(1.0277)(1.1214)(1.0127)\]

\[\Rightarrow R_{\text{year}} = 24.27\%\]

Q: What does this tell us about GE?

b. Solve for rate that sets PV of inflows equal to PV of outflows \(\Rightarrow NPV = 0\)

\[\Rightarrow \Rightarrow\text{ essentially solving for Internal Rate of Return (IRR)}\]

Notes:

1) this is not in the book and not in homework from the book, but there are some problems on old quizzes and exams
2) no assumption that reinvest dividends
3) outflows = purchase (or beginning) price of security
4) inflows = dividends (or other payments), sales (or ending) price of security

Ex.

<table>
<thead>
<tr>
<th>Date</th>
<th>Dividend</th>
<th>Price</th>
<th>Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31</td>
<td>$0.00</td>
<td>$15.13</td>
<td>0</td>
</tr>
<tr>
<td>2/25</td>
<td>$0.10</td>
<td>$15.92</td>
<td>56</td>
</tr>
<tr>
<td>6/17</td>
<td>$0.10</td>
<td>$15.91</td>
<td>168</td>
</tr>
<tr>
<td>9/16</td>
<td>$0.12</td>
<td>$16.23</td>
<td>259</td>
</tr>
<tr>
<td>12/22</td>
<td>$0.14</td>
<td>$18.06</td>
<td>356</td>
</tr>
<tr>
<td>12/31</td>
<td>$0.00</td>
<td>$18.29</td>
<td>365</td>
</tr>
</tbody>
</table>

\[NPV = -15.13 + \frac{.10}{(1 + r)^{365}} + \frac{.10}{(1 + r)^{168}} + \frac{.12}{(1 + r)^{259}} + \frac{.14}{(1 + r)^{356}} + \frac{18.29}{(1 + r)^{365}} = 0\]

\[\Rightarrow \text{Using Excel: } r = .2420 = 24.2\%\]

Q: What does this tell us about GE?
3. Average Annual Returns: \( \bar{R} = \frac{1}{T} \sum_{t=1}^{T} R_t \)  
(10.6)

where:

\( T = \) number of historical returns  
\( R_t = \) return over year \( t \)

\( \Rightarrow \) difficult to get your mind wrapped around a list of returns \( \Rightarrow \) need to summarize data

\( \Rightarrow \bar{R} \) equals the return would earn on average if invest year after year and distribution does not change

4. Variance and Volatility (Standard Deviation) of Returns:

\[ \text{Var}(R) = \frac{1}{T-1} \sum_{t=1}^{T} (R_t - \bar{R})^2 \]  
(10.7)

Note: dividing by \( T-1 \) rather than \( T \) gives unbiased estimator

Volatility = \( SD(R) = \sqrt{\text{Var}(R)} \)

\( \Rightarrow \) gives spread of possible returns

\( \Rightarrow \) the higher the volatility, the more spread out the returns

Ex. Based on the following annual returns on General Electric (GE) and General Mills (GIS), how did the average annual returns and volatility of GE compare to those of General Mills?

<table>
<thead>
<tr>
<th>Year</th>
<th>GE</th>
<th>GIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+1%</td>
<td>-2%</td>
</tr>
<tr>
<td>2</td>
<td>-64%</td>
<td>+12%</td>
</tr>
<tr>
<td>3</td>
<td>+39%</td>
<td>+25%</td>
</tr>
<tr>
<td>4</td>
<td>+29%</td>
<td>0%</td>
</tr>
<tr>
<td>5</td>
<td>-4%</td>
<td>+18%</td>
</tr>
<tr>
<td>6</td>
<td>+23%</td>
<td>+9%</td>
</tr>
</tbody>
</table>

\( \bar{R}_{GE} = \frac{1}{6} (1 - 64 + 39 + 29 - 4 + 23) = +4.00 \% \)

\( \bar{R}_{GIS} = \frac{1}{6} (-2 + 12 + 25 + 0 + 18 + 9) = +10.33 \% \)

Q: What do these two numbers tell us about GE and General Mills?
\[ \text{Var}(R_{\text{GE}}) = \frac{1}{5} \left[ (1 - 4)^2 + (-64 - 4)^2 + (39 - 4)^2 + (29 - 4)^2 + (-4 - 4)^2 + (23 - 4)^2 \right] = 138.16 \]

\[ \text{SD}(R_{\text{GE}}) = \sqrt{138.16} = 37.17\% \]

\[ \text{Var}(R_{\text{GIS}}) = \frac{1}{5} \left[ (2 - 10)^2 + (0 - 10)^2 + (12 - 10)^2 + (25 - 10)^2 + (18 - 10)^2 + (9 - 10)^2 \right] = 107.47 \]

\[ \text{SD}(R_{\text{GIS}}) = \sqrt{107.47} = 10.37\% \]

Q: What do the standard deviations tell us about GE and General Mills?
Q: Why would anyone invest in GE?

5. Standard Error (SE): Standard Deviation of Average

Notes:

1) the calculated average return is only an estimate of the true average
2) averages vary less than individual observations
3) the bigger our sample, the more confident we are in the average we calculated

=> Need some way to measure uncertainty about our estimate of the average return

Standard Error: \( SE = \frac{SD}{\sqrt{N}} \) \hspace{1cm} (10.8)

Where:

\[ SD = \text{standard deviation of the observations (individual returns)} \]
\[ N = \text{number of observations (size of sample)} \]

Ex. SE (Average return on GE) = 15.17\% = \( \frac{37.17}{\sqrt{6}} \)

6. Compound Annual Return

=> annual return required to duplicate the return on an asset over some period

\[ CAR = [(1 + R_1) \times (1 + R_2) \times \cdots \times (1 + R_T)]^{1/T} - 1 \]

Notes:

1) this is a geometric rather than an arithmetic average
2) the compound annual return is a better description of long-run past performance
3) the average annual return is the best estimate of an investment's expected return in the future.

Ex. Calculate the compound annual return on GE and General Mills (GIS) using the data from the previous example.

\[
\text{CAR(GE)} = \left[ (1.01)(0.36)(1.39)(1.29)(0.96)(1.23) \right]^{1/6} - 1 = -0.0427
\]

\[
\text{CAR(GIS)} = \left[ (0.98)(1.12)(1.25)(1.00)(1.18)(1.09) \right]^{1/6} - 1 = 0.0993
\]

II. Information, risk, and return (Review)

1. Types of News and Prices

   1) Firm-specific news: good or bad news about company itself
      Risk from firm-specific news called: firm-specific, idiosyncratic, unsystematic, unique, diversifiable risk

   2) Market-wide news: about economy and thus impacts all stocks
      Risk from market-wide news called: systematic, undiversifiable, market risk

2. Risk and Portfolios

   1) firm-specific risk diversifies away in large portfolios
   2) market risk does not
   3) volatility measures the risk of a well-diversified portfolio but not of an individual asset

3. Risk and Return

   1) investors will only earn a risk premium for systematic risk

      => no premium for firm-specific risk since diversifies away in a portfolio

   2) there is no clear relationship between average returns and volatility (standard deviation)