Chapter 3: Arbitrage and Financial Decision Making

Big picture:

1) If the price of an asset is less than its value, we should buy it.
2) If the price of an asset is greater than its value, we should sell it.
3) If we find two assets that always pay the same exact cash flows but which sell at two different prices, we should sell (short-sell) the high priced asset and buy the low priced assets.

Note: Only a handful of my former students have made a living finding and exploiting the type of mispricing in #3 above and then only for a few years

4) Since it is so easy to trade financial assets, we should not be able to find any of these three opportunities with market-traded securities. We might find such opportunities with real assets (land, factories, private companies, etc.)

Q: Why study something should not be able to do?

=> can use to figure out prices at which assets should trade

I. Financial Decision Making

A. Steps

1. Identify costs and benefits
   Note: work with accountants, managers, economists, lawyers, etc. to determine costs and benefits
2. Convert costs and benefits to equivalent dollars today
3. Proceed if the value of the benefits exceed the value of the costs

B. Equivalent Dollars (Value) Today

1. When competitive markets exist
   a. Definitions and example

   Competitive market: goods can be bought and sold at the same price

   Q: Do such markets exist?

   => The NYSE is pretty close

   => see price data on Ford on web
Bid price: highest price at which anyone is willing to buy
Ask price: lowest price at which anyone is willing to sell

Notes:

1) anyone can submit their own bid or ask price
   => called a limit order

2) anyone submitting a market order takes whatever price exists in the market now
   => if buying, they’ll pay whatever the ask price is (the lowest price that anyone is willing to sell for)
   => if selling, they’ll get whatever the bid price is (the highest price that anyone is willing to pay).

Q: What price will you pay if you submit a market order to buy/sell 100 shares of Ford?
Q: What happens if you submit limit order to buy/sell 100 shares of Ford?

b. Equivalent value today if competitive market: market price

Note: value doesn’t depend on individual preferences or expectations

Q: Assume your uncle gives you 100 shares of Ford. What is the gift worth?
Q: Would you trade your shares for $1000?
Q: Would you trade your shares for 100 shares of Honda?

2. When a competitive market does not exist

Note: This is when finance gets more interesting

Ex. Ford dealership on Hwy 84 in Waco (Bird Kultgen)

a. Equivalent value today: present value of future cash flows

Notes:

1) cash flows at different points in time are in different units
   => can’t combine or compare them
2) interest rate: exchange rate across time
   => allows us to convert dollars at one point in time to another point in time
3) interpretation: present value = amount would need to invest today at the current interest rate to end up with the same cash flow in the future
C. Making Decisions

1) Accept all positive NPV projects or the highest NPV project if must chose
2) NPV = present value of all cash flows (inflows and outflows)
3) Interpretation of NPV: wealth created by project
4) Another way to think about it: NPV equals the difference between the cost of the project and how much it would cost to recreate a project’s cash flows at the current interest rate

5) Decision doesn’t depend on preference for cash today vs. cash in the future

Ex. Assume you have an opportunity to buy land for $110,000 that you will be able to sell for $120,000 a year from today. Further assume that all cash flows are known for sure and that you can borrow or lend at a risk-free interest rate of 4%.

Q: Does a risk-free interest rate exist in the real world?

a. Should you buy the land if you have $110,000?

NPV = \(-110,000 + \frac{120,000}{1.04}\) = \(-110,000 + 115,384.60\) = 5384.6

Q: How much would you have to invest at 4% to end up with $120,000 a year from today?

=> $115,384.60

Q: How much better off are you if you buy the land? $5384.60

b. Should you buy the land if you have no money?

=> yes

Q: How?

=> borrow $115,384.60 and buy the land for $110,000

=> keep $5384.60 today

=> in one year sell the land and use the proceeds to pay off the loan

Q: Is this realistic?
II. Arbitrage and the Law of One Price

A. Introduction and Definitions

1. Equivalent assets: assets with exactly the same cash flows in all periods under all conditions

2. Arbitrage: trading to take advantage of price differences between equivalent assets trading in different markets

Note: requires no investment and creates riskless payoff

Ex. Assume the following prices for GE stock are available on the Boston Stock Exchange and the New York Stock exchange:

<table>
<thead>
<tr>
<th>Boston Stock Exch</th>
<th>New York Stock Exch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
<td>Ask</td>
</tr>
<tr>
<td>Price Size</td>
<td>Price Size</td>
</tr>
<tr>
<td>25.73 7000</td>
<td>25.76 6000</td>
</tr>
</tbody>
</table>

Arbitrage: simultaneously buy shares on the Boston Exchange and sell shares on the NYSE for risk-free profit of $720.

Note: Arbitrage profit = 6000 x (25.88 – 25.76)

Q: Why not trade more than 6000 shares?
Q: Why do we use $25.88 and $25.76?
Q: How long will these conditions last?

3. Normal market: no arbitrage possible

Reason should be “normal”: arbitrage will only exist until someone notices it…and a lot of people are looking for such opportunities.

4. Law of one price: equivalent assets trading at the same time in different normal markets must have the same price

5. Short sales:

1) today: borrow a security (usually from a broker) and sell it
2) later: buy same security and give it back to whoever you borrowed it from
Notes:

1) if the security has matured, might pay the cash value rather than buying the security and giving it back
2) must make up any cash flows the lender would have received while the security was borrowed
3) short seller can buy and return the security at any time
4) lender can demand the return of the loaned security at any time
5) it is important to distinguish between:
   a) buy: buy stock to create a long position
   b) buy to cover (a short position): buy stock and give back to lender of shares
   c) sell: sell to close out a long position
   d) short-sell: borrow and sell shares to create a short position

Ex. Assume you want to short-sell 100 shares of GE today for the market price of $12.50 per share

1) borrow 100 shares from your broker and sell them on the NYSE

   Q: Where stand?
   => owe your broker 100 shares of GE
   => have $1250 in your brokerage account

2) assume price falls from $12.50 to $10

3) Q: How close out short position?
   => buy 100 shares at $10 per share and give the shares to your broker

4) assume that while you were short GE paid a dividend of $0.10 per share
   => must give $10 to your broker.

5) Profit = +1250 – 10 – 1000 = $240

   Q: What would lead to a loss?

B. No Arbitrage Prices for Securities

   Key: For there to be no arbitrage, the price of any security must equal the present value of its cash flows

   Goal when setting up arbitrage: positive cash flow today, no possible net cash flow after today
Basic questions to ask when setting up an arbitrage:
1) What transaction (or set of transactions) is equivalent to the security?
2) Do you want to buy or sell the security?
3) What cash flows does this create?
4) What transaction today offsets the security’s cash flows in the future?

Ex. Assume you can borrow or lend at the risk-free rate of 7% and that a risk-free bond pays $1000 a year from today

\[ PV = \frac{1000}{1.07} = 934.58 \]

a) Assume price of bond is $920 (rather than its present value of $934.58)
=> arbitrage is possible

Q: What transactions today are equivalent to buying/short-selling the bond?

<table>
<thead>
<tr>
<th>Bond Position</th>
<th>Equivalent</th>
<th>Reason Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy bond</td>
<td>Lend $934.58</td>
<td>CF = +$1000 one year from today</td>
</tr>
<tr>
<td>Short-sell bond</td>
<td>Borrow $934.58</td>
<td>CF = – $1000 one year from today</td>
</tr>
</tbody>
</table>

Q: Buy or sell the bond?
Q: What are cash flows if buy bond?
Q: How end up with no cash flow next year?

Table 1

<table>
<thead>
<tr>
<th>Transaction (t=0)</th>
<th>$ today</th>
<th>$ in one year</th>
<th>Transaction (t=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy bond</td>
<td>-920.00</td>
<td>+1000.00</td>
<td>Payoff from bond</td>
</tr>
<tr>
<td>Borrow $934.58</td>
<td>+934.58</td>
<td>-1000.00</td>
<td>Pay off loan</td>
</tr>
<tr>
<td>Total</td>
<td>+14.58</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Arbitrage profit = $14.58

Table 1 Video

b) Assume price of bond is $950 (rather than its present value of $934.58)
Q: Buy or sell the bond?
Q: What are cash flows if short-sell bond?
Q: How end up with no cash flow next year?

Table 2

<table>
<thead>
<tr>
<th>Transaction (t=0)</th>
<th>$ today</th>
<th>$ in one year</th>
<th>Transaction (t=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-sell bond</td>
<td>+950.00</td>
<td>-1000.00</td>
<td>Buy to cover the bond</td>
</tr>
<tr>
<td>Lend $934.58</td>
<td>-934.58</td>
<td>+1000.00</td>
<td>Payoff on loan</td>
</tr>
<tr>
<td>Total</td>
<td>+15.42</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Arbitrage profit = $15.42

Table 2 Video

=> only way there is no arbitrage: price = $934.58
Notes:

1) investors rushing to take advantage of the arbitrage opportunity will quickly drive the price to $934.58

2) interest rates are usually extracted from security prices rather than the other way around

Ex. What is usually known: \( CF_1 = $1000, \) Price = $934.58

\[
934.58 = \frac{1000}{1 + r} \Rightarrow r = .07 = 7\%
\]

3) In a normal market, buying and selling securities has zero NPV

Keys:

a) NPV(buying security) = PV(CF) - price

\[
\Rightarrow \text{in normal market, price} = \text{PV(CF)} \Rightarrow \text{NPV} = 0
\]

b) NPV(selling security) = price – PV(CF)

\[
\Rightarrow \text{in normal market, price} = \text{PV(CF)} \Rightarrow \text{NPV} = 0
\]

\[
\Rightarrow \text{if price} \neq \text{PV(CF)}, \text{arbitrage is possible}
\]

Q: Why would anyone ever trade securities if NPV = 0?

C. No Arbitrage Prices of Portfolios

Key: In a normal market, equivalent portfolios (exactly same cash flows) must have same price

\[
\Rightarrow \text{otherwise arbitrage is possible}
\]

1. ETF: exchange traded fund

\[
\Rightarrow \text{essentially a portfolio of securities that you can trade on an exchange}
\]

2. Value Additivity: the price of a portfolio must equal the combined values of the securities in the portfolio

\[
\Rightarrow \text{Price}(A+B) = \text{Price}(A) + \text{Price}(B) \quad (3.5)
\]
Ex. Assume the following:

ETF1 holds one share of security A and one share of security B.
ETF2 holds asset C and asset D.
Security A pays $100 a year from today and has a market price of $95.24.
Security B pays $150 a year from today and has a market price of $142.86.
Asset C pays $200 a year from today and asset D pays $50 a year from today. Both are illiquid assets that are not traded in financial markets.

Q: What portfolio is equivalent to buying ETF1?

<table>
<thead>
<tr>
<th>Transaction</th>
<th>$ in one year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy A</td>
<td>+100.00</td>
</tr>
<tr>
<td>Buy B</td>
<td>+150.00</td>
</tr>
<tr>
<td>Total</td>
<td>+250.00</td>
</tr>
</tbody>
</table>

Q: Why is buying an A and buying a B equivalent to buying ETF1?

Q: What is the no-arbitrage price for ETF1?

=> 238.10 = 95.24 + 142.86

Reason: ETF1 must have the same price as a portfolio of A and B

Key to arbitrage with equivalent portfolios with different prices: buy low and sell high

Assume price of ETF1 is $220 instead of no-arbitrage price of $238.10

Arbitrage: Buy ETF1, short-sell equivalent portfolio

<table>
<thead>
<tr>
<th>Transaction (t=0)</th>
<th>$ today</th>
<th>$ in one year</th>
<th>Transaction (t=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy ETF1</td>
<td>-220.00</td>
<td>+250.00</td>
<td>Payoff on ETF</td>
</tr>
<tr>
<td>Short-sell A</td>
<td>+95.24</td>
<td>-100.00</td>
<td>Buy to cover A</td>
</tr>
<tr>
<td>Short-sell B</td>
<td>+142.86</td>
<td>-150.00</td>
<td>Buy to cover B</td>
</tr>
<tr>
<td>Total</td>
<td>+18.10</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 3

Table 3 Video
Assume price of ETF1 is $245 instead of no-arbitrage price of $238.10

Arbitrage: short-sell ETF1, buy equivalent portfolio

<table>
<thead>
<tr>
<th>Transaction (t=0)</th>
<th>$ today</th>
<th>$ in one year</th>
<th>Transaction (t=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-sell ETF1</td>
<td>+245.00</td>
<td>-250.00</td>
<td>Buy to cover ETF</td>
</tr>
<tr>
<td>Buy A</td>
<td>-95.24</td>
<td>+100.00</td>
<td>Payoff on A</td>
</tr>
<tr>
<td>Buy B</td>
<td>-142.86</td>
<td>+150.00</td>
<td>Payoff on B</td>
</tr>
<tr>
<td>Total</td>
<td>+6.90</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 Video

=> only way no arbitrage: price of ETF1 = 238.10
=> arbitrage will quickly drive the price of ETF1 to $238.10

Q: What does the market price for ETF2 have to be?

Note: payoff on ETF2 next year: 200 + 50 = 250

Q: What portfolio is equivalent to ETF2?

=> ETF1

=> must be worth 238.10

Reason: otherwise arbitrage is possible between ETF1 and ETF2

D. No-Arbitrage Pricing in an Uncertain World

1. Key ideas

   1) investors prefer less risk other things equal

      Reason: for most people a $1 loss is a bigger deal than a $1 gain

   2) Risk premium: extra return demanded by investors for holding risky assets instead of Treasuries

      => compensates investors for taking any risk

2. Risk premium on the market

   => the compensation for taking the market’s risk
Note: the market risk premium will increase if:

- the risk of the market increases or,
- investors become more risk averse

3. Risk premium on a security

Key => Depends on two things:

1) risk premium on market index
2) degree to which security’s return varies with market index.

=> the more it varies with the market, the higher the risk premium

Ex. Assume the following:
– risk-free interest rate = 2%
– a strong or weak economy is equally likely
– price of the market index: $100
– payoff on stock market index depends on the economy as follows:
  weak economy = $75
  strong economy = $139

– payoff on Orange Inc. depends on the economy as follows:
  weak economy = $95
  strong economy = $159

Q: What are the expected cash flow next year, the possible returns, the expected return, and the risk premium on the market?

=> expected cash flow for the market index next year = \( \frac{1}{2}(75) + \frac{1}{2}(139) = 107 \)

=> return on the market depends on the economy as follows:
  Strong: \( \frac{139 - 100}{100} = 39\% \)
  Weak: \( \frac{75 - 100}{100} = -25\% \)

=> expected return on the market index: \( \frac{107 - 100}{100} = 7\% = \frac{1}{2}(39\%) + \frac{1}{2}(-25\%) \)

=> risk premium on the market index = 5% = 7 – 2

Q: What is the no-arbitrage price of Orange Inc.?

Q: How does the payoff on Orange compare to the payoff on the market?

=> Orange always pays $20 more than the market
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Q: How create a portfolio that is equivalent to buying Orange?

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Weak</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy market index</td>
<td>+75.00</td>
<td>+139.00</td>
</tr>
<tr>
<td>Buy risk-free bond</td>
<td>+20.00</td>
<td>+20.00</td>
</tr>
<tr>
<td>Total</td>
<td>+95.00</td>
<td>+159.00</td>
</tr>
</tbody>
</table>

Cost to build a portfolio that is equivalent to Orange:

=> Cost of equivalent portfolio = 119.61 = 100 + \frac{20}{1.02} = 100 + 19.61

=> the price of Orange must equal 119.61 => otherwise arbitrage

Q: What is arbitrage profit if the price of Orange is $125?

<table>
<thead>
<tr>
<th>Transaction</th>
<th>$ today</th>
<th>Weak</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-sell Orange</td>
<td>+125.00</td>
<td>-95.00</td>
<td>-159.00</td>
</tr>
<tr>
<td>Buy market index</td>
<td>-100.00</td>
<td>+75.00</td>
<td>+139.00</td>
</tr>
<tr>
<td>Buy risk-free bond</td>
<td>-19.61</td>
<td>+20.00</td>
<td>+20.00</td>
</tr>
</tbody>
</table>
| Total                | +5.39   | 0.00  | 0.00

Table 5 Video

Q: What is the arbitrage profit if the price of Orange is $110?

<table>
<thead>
<tr>
<th>Transaction</th>
<th>$ today</th>
<th>Weak</th>
<th>Strong</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy Orange</td>
<td>-110.00</td>
<td>+95.00</td>
<td>+159.00</td>
</tr>
<tr>
<td>Short-sell market index</td>
<td>+100.00</td>
<td>-75.00</td>
<td>-139.00</td>
</tr>
<tr>
<td>Short-sell risk-free bond</td>
<td>+19.61</td>
<td>-20.00</td>
<td>-20.00</td>
</tr>
</tbody>
</table>
| Total                | +9.61   | 0.00  | 0.00

Table 6 Video

Q: What are the possible returns, expected return, and risk premium on Orange if it is correctly priced at $119.61?

Return on Orange if strong economy = 32.9% = \frac{159 - 119.61}{119.61}

Return on Orange if weak economy = -20.6% = \frac{95 - 119.61}{119.61}
Note: return on Orange less volatile than the market (+39% or -25%)

Note: The risk premium on Orange should be less than the market (5%).

Q: Why?

Expected cash flow for Orange = \( \frac{1}{2} (159) + \frac{1}{2} (95) = 127 \)

Expected return on Orange = \( \frac{127 - 119.61}{119.61} = \frac{1}{2} (32.9\%) + \frac{1}{2} (-20.6\%) = \frac{.062}{.02} = 6.2\% \)

Risk premium on Orange = \( .062 - .02 = .042 \)

E. Transaction cost: cost to trade securities

Note: transaction costs include:
1. commission to broker
2. bid-ask spread: difference between bid price and ask price
3. fees to borrow stock (varies with demand for shares to short)

Key: Transaction costs lead to the following modifications of earlier definitions:

Normal market => no arbitrage after transaction costs covered
Law of one price => difference in prices for equivalent securities must be less than transaction costs of engaging in arbitrage
No arbitrage price => differences between price and the PV(CF) must be less than transaction costs
Portfolio prices => Difference between the price of a portfolio and the sum of the prices of assets in the portfolio must be less than the transaction costs to build or break apart the portfolio