Instructions: Please clearly identify your solution with **bold** or by circling so that I can easily see your answer. Print your entire name at the top left of every page. I will not accept late assignments. Staple or paperclip your answers to this handout. Write legibly.

Use the following production functions to answer questions 1-3:

\[
q(K, L) = K^{\frac{1}{3}} L^{\frac{2}{3}} \\
q(K, L) = \left( \frac{K^{-2}}{2} + \frac{L^{-2}}{2} \right)^{\frac{1}{2}}
\]

1. The following questions primarily reference the marginal rate of technical substitution (MRTS):

   (a) Derive the MRTS for each production function (assuming it exists) and evaluate whether it is diminishing. Rigorously verify your answer.

   (b) What are the returns of scale of each production function? Verify your answer.

   (c) Find the function, or the value of, the elasticity of substitution for each production function.

2. Find the conditional input demand functions, \( K(r, w, q) \) and \( L(r, w, q) \), and total cost function, \( C(r, w, q) \), associated with each production function.

   (a) Legislators pass a bill raising the minimum wage over the equilibrium wage. Rigorously show the long-run effect on both labor demand and capital demand. Are capital and labor substitutes or complements? Does the law of demand for labor hold for this production function?

   (b) Rigorously evaluate the effect of the minimum wage law on total costs of production