Use each of the utility functions below to answer the following questions. Always assume $x, y > 0$:

(a) $U = f(x, y) = x^{\frac{1}{4}}y^{\frac{3}{4}}$
(b) $U = f(x, y) = x^{\frac{1}{2}} + y^{\frac{1}{2}}$
(c) $U = f(x, y) = y - x^{-1}$

1. Derive the marginal rate of substitution ($MRS_{y,x}$) functions. Is $MRS$ diminishing?
   Does each of these three utility functions represent unique preferences? Why or why not?

2. Derive the uncompensated (Marshallian) demand functions for $x(P_x, P_y, I)$ and $y(P_x, P_y, I)$.\(^1\)

3. Are the goods normal or inferior? Verify your answer.

4. Derive the indirect utility function, $V(P_x, P_y, I)$.

\(^1\) $P_x$ and $P_y$ are the market prices of good $x$ and $y$, and $I$ is exogenous consumer income. Assume all prices and income are non-negative.