Shipping the Good Apples Out: The Alchian and Allen Theorem Reconsidered

Thomas E. Borcherding
Simon Fraser University

Eugene Silberberg
University of Washington

Gould and Segall argued that the introduction of a third, composite good vitiates the Alchian and Allen (A-A) theorem that a common charge on two substitute goods leads, real income held constant, to a relative increase in the consumption of the higher to lower quality commodity. Using Hicks's third law, however, it is demonstrated that the direct substitution effect tends to dominate the interaction effect with the third good, if the two substitutes are close. Absolute changes are also investigated and some operational propositions offered with casual supporting observations. A-A's proposition is shown to be a useful price-theoretic construction, though not a direct implication of the law of demand.

One of the more interesting empirical generalizations that has appeared in recent years is the proposition that if the same fixed cost, for example, a transport cost or a "per item" tax, is added to the prices of two similar goods, the effect will be to raise the relative consumption of the higher quality or premium good. Though apparently a part of the UCLA oral tradition for many years, it first appeared in print in Armen Alchian and William Allen’s text, University Economics (1964, pp. 74–75).1

I. A Casual Empiricism

A striking example of the phenomenon is contained in the following letter, by an irate consumer, to the “Troubleshooter” column of the

We have benefited from conversations with and comments by Armen Alchian, Yoram Barzel, Stephen Cheung, Robert Deacon, and Erwin Diewert. Particularly helpful were the suggestions of John Gould. Any remaining errors are of course our own.

1 Since then the proposition also appears as a problem in Stigler’s (1966, p. 103) and Hirshleifer’s (1976, p. 321) respective texts.

© 1978 by The University of Chicago. 0022-3808/78/8601-0008$00.81.

131
Seattle Times (October 19, 1975): “Why are Washington apples in local markets so small and old-looking? The dried-up stems might seem they were taken out of cold storage from some gathered last year. Recently, some apple-picking friends brought some apples they had just picked, and they were at least four times the size of those available for sale here. Where do these big Delicious apples go? Are they shipped to Europe, to the East or can they be bought here in Seattle?—M. W. P.” An answer from a trade representative (Seattle Times, October 19, 1975) allowed that “itinerant truckers” (price cutters) were at fault: “... [T]he apples [she] is seeing in her local markets may have been some left from the 1974 crop, or could be lower-grade fruit sold store-to-store by itinerant truckers.” The textbook answer was supplied by one of the authors several days later (Seattle Times, October 28, 1975):

Comparing Apples to Apples

Reaction: “Regarding M. W. P.’s complaint (Sunday, October 19) that all the good apples were being shipped to the East, you might be interested to know that ‘shipping the good apples out’ has been a favorite classroom and exam question in the economics department at U.W. for many years.

“It is a real phenomenon, easily explained:

“Suppose, for example, a ‘good’ apple costs 10 cents and a ‘poor’ apple 5 cents locally. Then, since the decision to eat one good apple costs the same as eating two poor apples, we can say that a good apple in essence ‘costs’ two poor apples. Two good apples cost four poor apples.

“Suppose now that it costs 5 cents per apple (any apple) to ship apples East. Then, in the East, good apples will cost 15 cents each and poor ones 10 cents each. But now eating two good apples will cost three—not four poor apples.

“Though both prices are higher, good apples have become relatively cheaper, and a higher percentage of good apples will be consumed in the East than here.

“It is no conspiracy—just the [law of demand].”

This appealing line of argument was challenged by John Gould and Joel Segall, who demonstrated that in a three-good world Alchian and Allen’s substitution postulate does not follow from the law of demand, since interactions with the third good might destroy the effect (Gould and Segall 1968). As contrary evidence, Gould and Segall asked, rhetorically: “How often is it heard, for example, that the way to get really good farm produce is to drive out to the country and buy it at a roadside stand or that one must go to Maine to get truly delectable lobsters?”

Our purpose in this article is twofold: first, to show that even if accepted as true, Gould and Segall’s casual empirical observations in
no way contradict Alchian and Allen’s proposition (and even support it), and, second, by reformulating the model in a more tractable fashion, to show that a deeper analysis indicates that the above proposition while indeed not a mathematical consequence of the law of demand is apt to be true in the circumstances in which it was meant to apply.

The Alchian and Allen proposition assumes that nothing happens to the goods themselves as a result of the price changes. In the transportation charge formulation (which we shall temporarily maintain), nothing is supposed to occur to the goods, that is, no spoilage, ripening, or other quality changes en route to the final destination. To the extent, therefore, that Gould and Segall’s examples depend on spoilage of the produce or lobsters, these observations are not counterexamples to Alchian and Allen’s proposition. More important, however, it does not matter if the goods are shipped to the consumers or the consumers are shipped to the goods. Going to Maine, or to the country, involves a transport cost to people not from Maine or the country. What the above proposition predicts, therefore, is that tourists in Maine will consume, on average, higher quality lobsters than the natives and similarly for city versus rural dwellers’ purchases of produce at roadside stands. The existence of roadside stands specializing in high quality produce would therefore confirm, not refute, Alchian and Allen. Similarly, if people who make a special trip to Maine in fact choose to eat “truly delectable” instead of inferior quality lobsters sold there, this confirms Alchian and Allen’s thesis. Going to Maine to eat lobsters therefore in no way contradicts the proposition under analysis here.²

II. The Theoretical Model

Let us now turn to the theoretical development of the model. Let us assume that there are three goods, \( x_1 \), \( x_2 \), and \( x_3 \), respectively. The good \( x_3 \) shall be considered to be a Hicksian composite commodity, representing “all other goods.” Let \( x_1 \) and \( x_2 \) be, respectively, the “superior” and “standard” qualities of some good, so that \( p_1 > p_2 \).

For each of these commodities there is a compensated demand curve, that is, a demand curve which holds utility, or “real” income, constant:

\[
\begin{align*}
  x_1 &= x_1^U(p_1, p_2, p_3, U) \\
  x_2 &= x_2^U(p_1, p_2, p_3, U) \\
  x_3 &= x_3^U(p_1, p_2, p_3, U)
\end{align*}
\]  

(1)

² Going to Maine in a chartered jet and then eating the lower quality lobsters offered for sale there would contradict Alchian and Allen; however, no such observation has been provided.
We shall consider only compensated changes in consumption, since, as Gould and Segall point out, income effects are always indeterminate and, if strong enough, can destroy this or virtually any other proposition in economics.

Suppose now that a transport cost, \( t \), per item, is added to the prices of \( x_1 \) and \( x_2 \). The prices at the distant location therefore become \( p_1 + t \), \( p_2 + t \), \( p_3 \), respectively. Alchian and Allen’s thesis can therefore be stated as:

\[
\frac{\partial(x_1 \div x_2)}{\partial t} > 0. \tag{2}
\]

That is, as the transport cost rises, the higher quality good increases in consumption relative to the lower quality good, holding real income and the prices of all other goods constant.

It should be pointed out that the addition of a constant amount \( t \) to \( p_1 \) and \( p_2 \) is predicated upon certain assumptions. For this to be a valid procedure, it must be assumed that among all the attributes that comprise \( x_1 \) and \( x_2 \), there must be one measurable characteristic common to both, to which the transport cost (or other common change) is applied. Exactly when the other attributes are sufficiently similar so that the consumer will regard the goods as two qualities of the same good as opposed to two different goods seems arbitrary. A higher quality good presumably possesses some attributes that the consumer finds desirable which are found in greater amounts than in the lower quality good (or perhaps it contains less of some undesirable attribute). For the ensuing analysis to be valid all that is required, as will be shown below, is that the market determines that two goods are different grades of one class of commodities, that is, they are close substitutes, and that one of these two so identified goods is the more expensive.

It is immediately apparent that relation (2) is not derivable from the law of demand in a three-good world. The law of demand is a proposition about how the demand for one good responds to a change in one relative price (holding income constant). Here, however, \( p_1 \) and \( p_2 \) are both changing relative to \( p_3 \). With two price changes, the law of demand is inapplicable. This is why Gould and Segall were able to demonstrate that Alchian and Allen’s proposition was not implied by the usual economic postulates. Their analysis is entirely correct on this point.

However, it is possible to delve deeper into these matters by expanding the quotient in equation (2):

\[
\frac{\partial(x_1 \div x_2)}{\partial t} = (x_2 \partial x_1 \div \partial t - x_1 \partial x_2 \div \partial t)(1 \div x_2^2). \tag{3}
\]

An increase in \( t \) is equivalent to increasing \( p_1 \) and \( p_2 \) by the same amount, hence \( \partial x_i \div \partial t = \partial x_i \div \partial p_1 + \partial x_i \div \partial p_2 \), \( i = 1, 2 \). Letting \( s_{ij} = \partial x_i \div \partial p_j \), the

\[\text{Superscripts will be dropped for notational ease.}\]
Hicksian pure substitution terms, equation (3) becomes
\[ \frac{\partial(x_1/x_2)}{\partial t} = (x_1/x_2) \left( \frac{s_{11}}{x_1} + \frac{s_{12}}{x_1} - \frac{s_{21}}{x_2} - \frac{s_{22}}{x_2} \right). \] (4)

Defining the compensated elasticities \( e_{ij} = \left( \frac{p_j}{x_i} \right) \left( \frac{\partial x_i}{\partial p_j} \right) \), equation (4) becomes
\[ \frac{\partial(x_1/x_2)}{\partial t} = (x_1/x_2) \left( \frac{\varepsilon_{11}}{p_1} + \frac{\varepsilon_{12}}{p_2} - \frac{\varepsilon_{21}}{p_1} - \frac{\varepsilon_{22}}{p_2} \right). \]

From Hicks's third law (1946, pp. 310–11),
\[ \sum_{j=1}^{3} e_{ij} = 0, \quad i = 1, 2, 3. \]

Using this expression to substitute for \( e_{12} \) and \( e_{22} \) in equation (4), one obtains
\[ \frac{\partial(x_1/x_2)}{\partial t} = (x_1/x_2) \left[ \frac{\varepsilon_{11}}{p_1} + \left( -\varepsilon_{11}/p_2 - \varepsilon_{13}/p_2 \right) - \varepsilon_{21}/p_1 \right. \\
\left. - \left( -\varepsilon_{21}/p_2 - \varepsilon_{23}/p_2 \right) \right] \]

or
\[ \frac{\partial(x_1/x_2)}{\partial t} = (x_1/x_2) \left[ (\varepsilon_{11} - \varepsilon_{21}) (1/p_1 - 1/p_2) + (\varepsilon_{23} - \varepsilon_{13}) (1/p_2) \right]. \] (5)

Equation (5) offers some insights into the Alchian and Allen hypothesis not explored previously. If \( x_1 \) and \( x_2 \) are substitutes, \( e_{21} > 0 \), and, of course, \( e_{11} < 0 \). Moreover, since \( x_1 \) is the premium good, \( p_1 > p_2 \), or \( 1/p_1 < 1/p_2 \). Thus, the first term in the square brackets above, the direct substitution effect, must be positive. In a two-good world, this would be the entire expression for \( \frac{\partial(x_1/x_2)}{\partial t} \) and would be the Alchian and Allen thesis. In a three-good world, the last term, the interaction effect of \( x_1 \) and \( x_2 \) with \( x_3 \), in particular \( (\varepsilon_{23} - \varepsilon_{13}) \), comes into play. This latter term is mathematically indeterminate. As an empirical matter, however, it seems that this term might often be expected to be dominated by the first term.

If \( x_1 \) and \( x_2 \) are assumed to be close substitutes, there is little reason to presume that their interactions with the composite commodity should be widely disparate. We should be surprised if the cross elasticities of Golden Delicious and McIntosh apples with other goods differed widely. Thus, the term \( (\varepsilon_{23} - \varepsilon_{13}) \) should be small. Moreover, as \( x_1 \) and \( x_2 \) become closer and closer substitutes, \( e_{11} \) and \( e_{12} \) become unboundedly large in absolute value (though of opposite sign), making the first term in equation (5) tend to \( +\infty \). However, \( e_{13} \) and \( e_{23} \) not only remain bounded, they must tend to equality as \( x_1 \) and \( x_2 \) become even closer substitutes. Thus, when \( x_1 \) and \( x_2 \) are close substitutes, we should expect to see Alchian and Allen’s hypothesis confirmed. The effect would be confounded if, say, the premium good is a close substitute to the composite commodity (\( e_{13} > 0 \)), and yet the standard good is a complement to the third good...
We find this an implausible circumstance. Even if true, it is still the case that the first term will swamp the second if $x_1$ and $x_2$ are very close substitutes, for the above stated reasons.

A similar result can be derived from the difference, as opposed to the ratio of consumption of $x_1$ to $x_2$, when $t$ changes. Letting $p_1 = p_2 + k$, $k > 0$, from Hicks's third law, again:

$$(p_2 + k)s_{11} + p_2 s_{12} + p_3 s_{13} = 0$$

$$(p_2 + k)s_{21} + p_2 s_{22} + p_3 s_{23} = 0.$$ 

Since $\partial x_1/\partial t = s_{1t} = s_{11} + s_{12}$, $\partial x_2/\partial t = s_{2t} = s_{21} + s_{22}$,

$$p_2 s_{1t} + ks_{11} + p_3 s_{13} = 0$$

$$p_2 s_{2t} + ks_{21} + p_3 s_{23} = 0.$$ 

Subtracting,

$$p_2(s_{1t} - s_{2t}) = -k(s_{11} - s_{21}) + p_3(s_{23} - s_{13}). \quad (6)$$

The additive analogue of Alchian and Allen's thesis is that $s_{1t} > s_{2t}$. We note that under the same assumptions as before, the first term on the right-hand side of equation (6) is positive, confirming the result for the case of two commodities. With three commodities, it is the ordinary rates of change (as opposed to elasticities) of $x_1$ and $x_2$ with respect to changes in $p_3$ that matter, since the difference $(s_{1t} - s_{2t})$ is being analyzed. It is more difficult to comment on the size of $(s_{23} - s_{13})$; we can note that as before, as $x_1$ and $x_2$ become closer and closer substitutes, $(s_{11} - s_{21}) \rightarrow -\infty$ while $(s_{23} - s_{13})$ remains bounded. Hence, for close substitutes (and, again, this is the intended application of this theorem), the premium good should rise in consumption relative to the lower quality good, in an additive sense, if the same transport cost is added to both goods.

III. Conjectures and Conclusions

The proposition that the addition of per item costs will cause the higher quality goods to be consumed in relatively greater amounts is in fact not derivable from the curvature properties of utility functions alone, as Gould and Segall pointed out. We feel, however, that it would be incorrect to conclude that this proposition is therefore not useful in economics. The law of demand is not implied by utility maximization; it is still useful. In applying these empirical generalizations, certain test conditions must be maintained; most notably, income must be held constant. In addition, for Alchian and Allen's proposition, the goods
must be assumed to be close substitutes. Thus, for example, if the distinguishing attributes of produce are time between harvest and consumption, or for lobster, size and time since capture, these operational measures can be used as proxies for grade and for assuming high cross elasticities. These assumptions are, in principle at least, capable of independent confirmation, and thus Alchian and Allen’s substitution theorem has predictive content.

In addition, the proposition has application well beyond the transportation cost scenario. The analysis applies when any kind of cost item is added equally to similar goods. Consider, for example, the amount of fine tailoring that will be done on clothing made from expensive rather than cheap fabric. We can expect the expensive fabric to be more carefully and elaborately tailored, because tailoring is relatively cheaper on expensive rather than inexpensive fabric. Consider also that most top grade (e.g., USDA “prime”) beef is sold to restaurants, where the relative cost of consuming such beef is lower than at home, given the cost of cooks, waiters, fancy decor, etc. Houses situated on lots with high site value tend to be fancier (and more expensive) than those situated in places with low site value.4

Even the definition of a commodity itself reflects Alchian and Allen’s proposition. Consider that produce is more apt to be sold by weight rather than by the piece in the winter than in the summer. The absolute cost of measuring the goods is presumably constant throughout the year. In season, however, produce is relatively cheap, and hence, the metering costs are relatively high, thus less metering is done. A more extreme version of this phenomenon is provided by the measurements done on diamonds as opposed to less valuable jewels. Since diamonds are very high priced relative to rhinestones, the cost of measuring diamonds is relatively low. Diamonds are, in fact, measured extensively, with regard to color and cut as well as by weight (to several decimal places of carats) and even with regard to flaws not visible to the naked eye. Rhinestones are never measured this extensively, since these same absolute measurement costs are relatively higher for rhinestones than diamonds.

None of the above phenomena, strictly speaking, is a direct consequence of the law of demand, even if real income is held constant. All need the additional assertion that the terms \((e_{23} - e_{13})\) [or \((s_{23} - s_{13})\)] are positive, or small relative to the first term in equations (5) or (6), respectively. Yet the above observations seem to us to be pervasively true, as an empirical matter. Alchian and Allen’s “indirect evidence of validity” (of the law of demand) provides perhaps some of the most interesting and reliable predictions in economics.

---

4 The hilly topography of Seattle or Vancouver’s north shore provides dramatic confirmation of this prediction.
References


