# Notes on the Theory of Intertemporal Choice:

## The Case of Certainty

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## I. Introduction

In <u>The Theory of Interest</u>, Irving Fisher developed a model of intertemporal choice by individual economic agents. The decisions of primary concern to Fisher were the consumption, investment-savings, and borrowing-lending relationships in the context of perfect certainty. Having developed the model for individuals, Fisher aggregated across individuals to develop a theory of financial markets and an explanation of the process by which the equilibrium market rate of interest is determined.

Our task in these notes is a much more modest undertaking. While the interest rate determination process is of some interest to us, we are more concerned with individual choice processes. We ultimately wish to derive decision rules for capital investment decisions by firms. Further, we assume that firms attempt to act in the "best interest" of its owners who contribute the funds to finance the capital investments. This suggests several questions which the manager might want to have answered:

1. What is the "best interest" of owners and how is it measured?
2. How should owners' preferences be taken into account in making investment decisions?
3. When owners have diverse preferences, whose preferences should dominate? major stockholder's? majority coalition's? managers'?
4. Is there some objective rule which may be followed which does not depend on knowledge of individual preferences?

It should be clear at this point that a detailed study of the individual and his allocation process between consumption and investment is necessary before attempting to solve the capital allocation problem of the firm.

We shall undertake the model of the individual in two sequential approximations. In the first we suppose the individual has only market opportunities for transforming his income streams into alternative consumption patterns. Stated more simply, if the individual found himself with an abundance of income today he might wish to loan out the excess at interest for one period, thereby enhancing consumption next period. In the reverse case, he might augment current income by borrowing to be repaid with interest out of next period's income. In either case, the borrowing and lending are purely financial transactions.

In the second approximation we introduce the opportunity for transformation between time periods through real investment. Accordingly the individual may allocate part of his current income to produce more income in the subsequent period. In addition he may borrow and lend to further transform his produced income streams into a preferred consumption pattern.

## II. First Approximation: Pure Exchange Model

We assume for our purposes that the individual's horizon is one period long. Hence there are two discrete points in time: "now" referred to by "time zero" or $t = 0$ and the "end of the period" referred to by "time one" or $t = 1$. The time lapse between $t = 0$ and $t = 1$ is of the indefinite length of "one-period" although it may be convenient to think of this period as one year long.

Each agent is endowed with income in each period from an undefined source. The endowment is shown in Fig. 1 by the point Y which consists of current income $y_0$ and future income $y_1$. Notice that the axes are measured in terms of $c_0$, current consumption, and $c_1$, future consumption. We are not concerned here with what particular commodities are consumed in any period but simply with how much is consumed in each period. Hence, it is sufficient to think of consumption units as an index of how many "bundles" of commodities are consumed in a period. Thus $c_1$ units differ from $c_0$ units only in that they are consumed at different times so they are often called "dated consumption units." Also $y_0$ and $y_1$ are measured in terms of these bundles and are sometimes called "dated consumption claims."

We suppose individuals derive satisfaction, or "utility," from consuming combinations of $c_0$ and $c_1$ where "consuming" simply means having the right to control the commodities $c_0$ and $c_1$. (For example, if the individual gives a unit to a friend he is still said to "consume" the unit.) Further, we suppose the individual can compare alternative combinations of $c_0$ and $c_1$. Given two alternatives $A = (c_0^A, c_1^A)$ and $B = (c_0^B, c_1^B)$ he could conclude one of three possibilities:

1. A is preferred to B,
2. B is preferred to A, or
3. A and B are equally preferred.

If certain mathematical requirements are satisfied we could construct a preference index, u, often called a utility function, of the general form

$$u = u(c_0, c_1).$$

The function u need only be ordinal to preserve the rankings possessed by the individual.

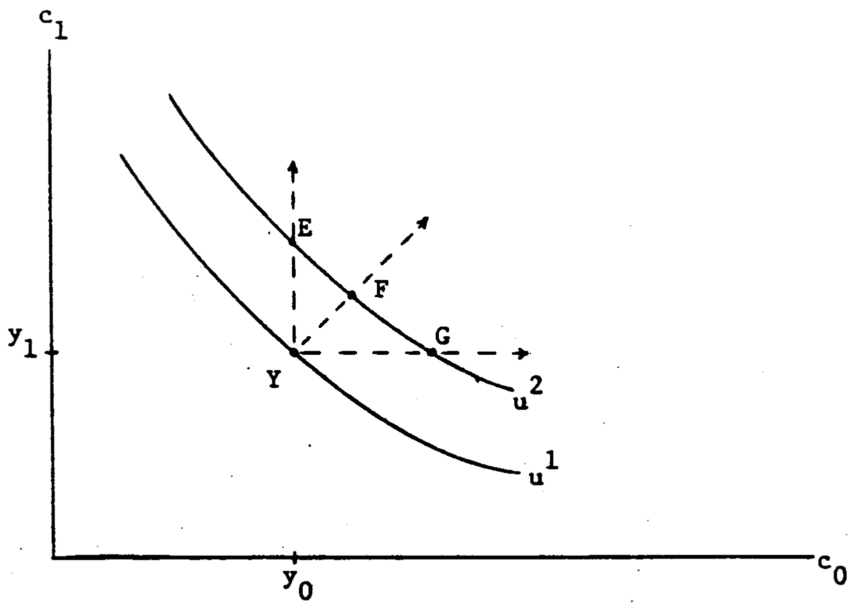


Fig. 1: Endowment and the Family of Indifference Curves

Usually there are numerous combinations of $(c_0, c_1)$ among which the individual is indifferent. In Fig. 1 we have traced out these alternative combinations along the curve $u^1$ which is called an "indifference curve." The individual would be indifferent between consuming his endowment and any other point on $u^1$. All such points on $u^1$ would give the same value for the utility index u.

We assume individuals prefer more to less which means that point E is preferred to the endowment Y because it offers <u>more</u> future consumption and the <u>same</u> current consumption. Similarly point G is preferred because it has <u>more</u> $c_0$ with the <u>same</u> $c_1$ and F has <u>more</u> of each. Since E, F, and G are on $u^2$ they are equally preferred and the index for $u^2$ exceeds that for $u^1$. In general, any point above $u^1$ is preferred to points on $u^1$, points below $u^1$ are less preferred, and points on $u^1$ are equivalent.

We could define a whole family of curves like $u^1$ and $u^2$. We assume these curves to be convex to the origin and any movement to the "northeast" is preferred.

Another important property of the indifference curve is its slope. Consider the following conceptual experiment. Suppose we offer the individual with endowment Y in Fig. 2 a chance to give up one unit $c_0$ in exchange for additional $c_1$. What is the <u>minimum</u> amount of $c_1$ that he would accept without being any worse off? Clearly we are talking about movements along $u^1$ that leave him equally well off. Fig. 2 shows several successive movements.

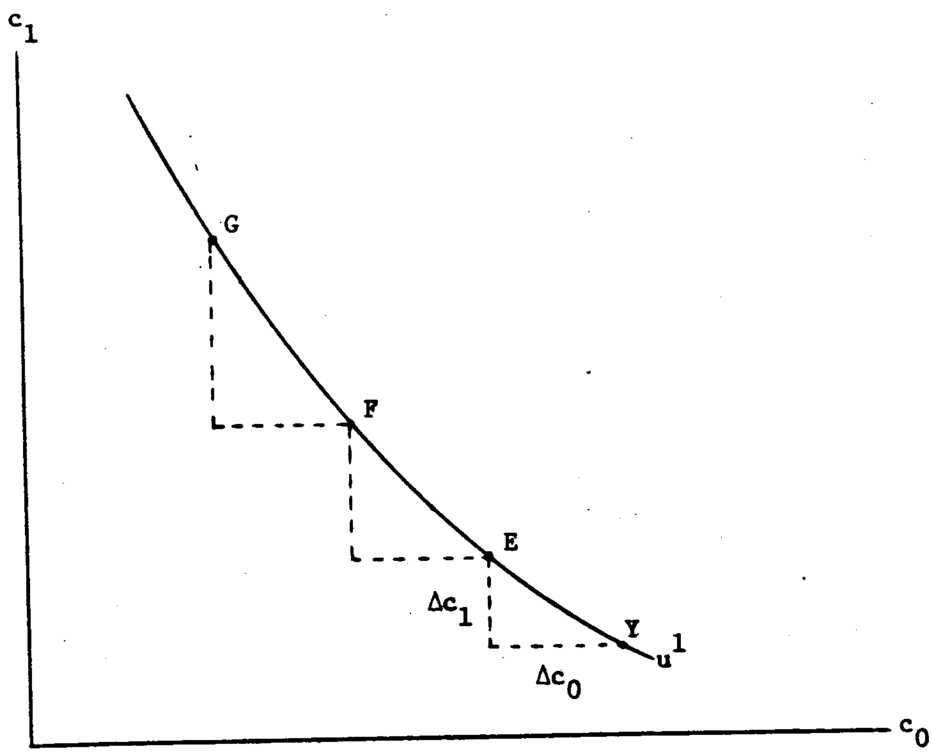


Fig. 2: Rates of Substitution

In the movement from Y to E we see the individual is quite willing to exchange $c_0$ for $c_1$ since he is relatively well endowed with $c_0$ but less well endowed with $c_1$. On the other hand in moving from F to G we see he requires a much larger amount of $c_1$ for the sacrifice of the same amount of $c_0$ since he is relatively well stocked with $c_1$. This will always be the case as we move up the indifference curve.

A meaningful ratio is the rate of the substituion or $\Delta c_1/\Delta c_0$ which is negative since $\Delta c_1$ and $\Delta c_0$ have opposite signs. That is, $c_0$ is decreasing but more $c_1$ is being substituted. This measure is analogous to the concept of slope which is defined at a point by a line tangent to $u^1$. This slope is called the "marginal rate of substitution" or simply MRS. It reflects his willingness to trade-off $c_0$ for $c_1$. As we move up $u^1$ the curve becomes steeper which reflects a diminished willingness to give up yet another unit of $c_0$ for more $c_1$ unless the rate of exchange is increased.

Since MRS is negative, it is confusing to speak of it as "increasing" (moving closer to zero) and "decreasing" (moving more negative) so we introduce the notion of the "rate of time preference" or rtp where

$$MRS = -(1 + rtp).$$

We can think of three cases:

1. Impatience: $MRS < -1$, $rtp > 0$,
2. Neutrality: $MRS = -1$, $rtp = 0$, and
3. Patience : $MRS > -1$, $rtp < 0$.

The rtp is a measure of the premium <u>in terms of $c_1$</u> that must be offered to persuade the individual to give up a unit of $c_0$. For example, suppose $MRS = -1.3$. Then the individual requires 1.3 units of $c_1$ to give up 1.0 units of $c_0$. Thus $rtp = .3 = 30\%$ so he requires a 30% premium for willingly deferring a unit of current consumption

In general, rtp depends on the amounts of $c_0$, $c_1$ and the ratio $c_1/c_0$. In Fig. 3 we have drawn a $45^o$ line tangent to $u^1$ at point E.

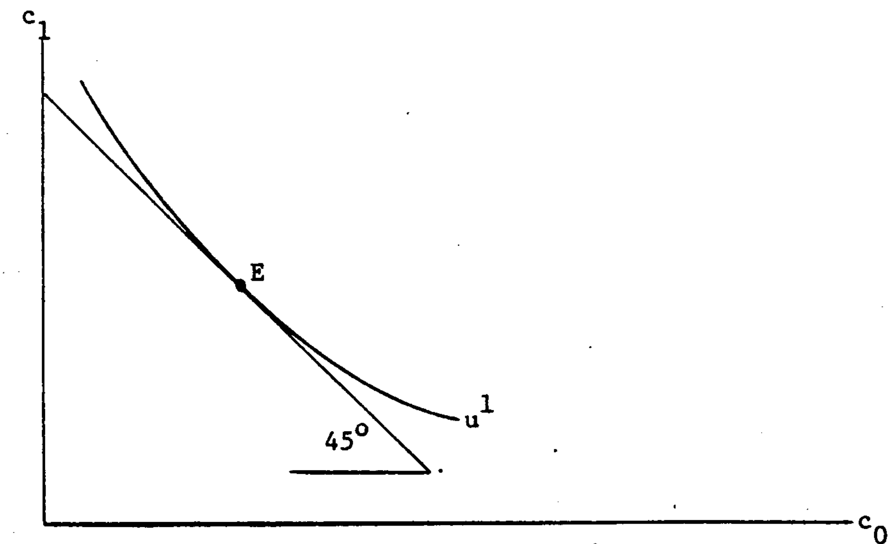


Fig. 3: Impatience and rtp

Since the slope is -1 at point E, MRS = -1 and rtp = 0 representing time neutrality. As we move up u' from point E we increase the relative proportion $c_1/c_0$ and increase rtp or impatience. The reverse holds as we move down $u^1$. (What is the relative endowment of a recent college graduate and what does this imply about his rtp and willingness to borrow? Is this consistent with observation? What about someone about ready to retire and his willingness to lend, i.e., save?)

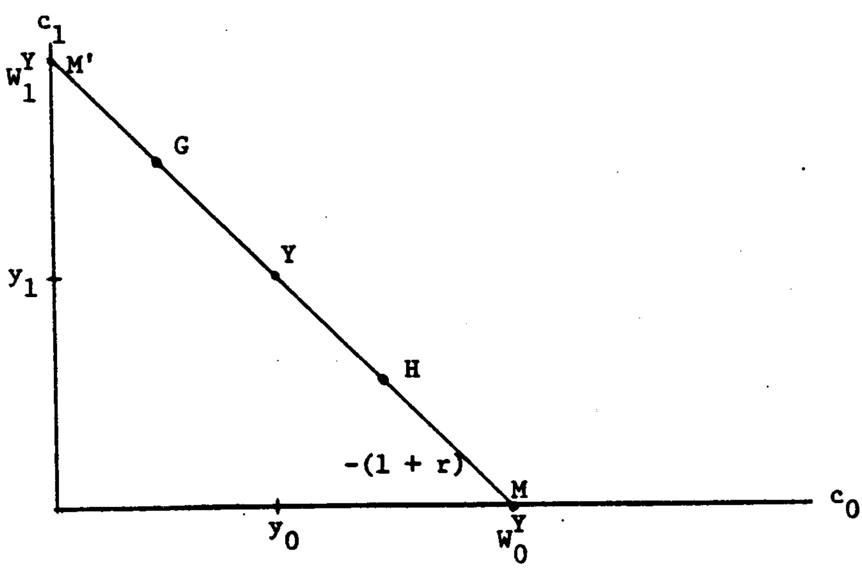


Fig. 4: Market Opportunities

So far we have mentioned only individual tastes reflected by rtp without introducing opportunities to exchange $c_1$ and $c_0$ through borrowing and lending. We now consider these latter possibilities along the line MM' in Fig. 4. Again the individual has endowment Y and he can trade his endowment for the consumption combination H by borrowing (augmenting $c_0$ at the sacrifice of $c_1$) or combination G by lending (augmenting $c_1$ by deferring part of $c_0$).

The slope of MM' is called the "marginal rate of exchange" or MRE. Since it is negative we may define it as

$$\text{MRE} = -(1 + r)$$

where r is the "rate of interest." Thus r is the premium one can obtain for lending, i.e., deferring consumption. If the individual loans $\Delta c_0 = -1$ he can consume the increment $\Delta c_1 = (1 + r)$ next period. Alternatively, if he promises to give up $\Delta c_1 = -1$ he can consume $\Delta c_0 = 1/(1 + r)$ more now. Thus r is often called the "reward for waiting" or "the price of earlier availability." We can also speak of $\phi_0^1$ as the price of a unit of $c_0$ (hence the subscript) in terms of units of $c_1$ (hence the superscript) where

$$\phi_0^1 = (1 + r)$$

and $\phi_1^0$ as the price of a unit of $c_1$ in terms of $c_0$ units where

$$\phi_1^0 = 1/(1 + r).$$

(Of course, $\phi_0^0 = \phi_1^1 = 1$.) The relationship of the two prices is

$$\phi_0^1 = 1/\phi_0^1 .$$

We often call $\phi_0^1$ the "compound factor" and $\phi_1^0$ the "discount factor."

In Fig. 4 suppose Y = (100,100), i.e., $y_0 = 100$ and $y_1 = 100$, and $r = .1 = 10\%$. The individual might borrow 50 by promising to repay 55 and consume the combination (150,45) say at point H. Alternatively he could lend 40 now and obtain the combination (60, 144) as at point G. In fact, the individual may convert his endowment Y into any consumption combination along MM'. One such combination of special interest is the horizontal intercept. If the individual were to convert all future consumption to present consumption, the maximum amount he could consume currently is represented by point M or the combination $(W_0^Y, 0)$ where

$$W_0^Y = y_0 + \phi_1^0 y_1 = y_0 + \frac{y_1}{1 + r} .$$

While we should be surprised to observe such an exchange, the value of $W_0^Y$ has the interpretation of "wealth." That is to say, endowed wealth $W_0^Y$ is the present value of the individual's income. (Note that we could define $W_1^Y = y_0(1 + r) + y_1$ as "future wealth" but since the two measures are related by $W_0^Y = W_1^Y/(1 + r)$ we will use the standard notion of wealth.) In the present example of Y = (100,100) the wealth of the individual is 190.10 units of current consumption.

One additional aspect of Fig. 4 is worth mentioning. Since all points on MM' have present value of $W_0^Y$, then as the individual borrows or lends his current wealth remains <u>unchanged</u>. That is to say, borrowing and lending are purely <u>financial</u> (as opposed to <u>real</u>) transformations of his endowed income into alternative consumption patterns and do not change the amount of total consumption as measured in terms of current units. Further, with initial endowment and the rate of interest (which defines the slope) we may uniquely identify the market opportunity line MM' of potential consumption patterns. The individual is <u>constrained</u> by his wealth to select a point on MM'. (Why wouldn't he pick points below MM'?) Stated more formally, the present value of consumption is constrained by his income such that

$$c_0 + c_1/(1 + r) = y_0 + y_1/(1 + r) = W_0^Y .$$

(Incidentally, what happens to his consumption opportunity set and current wealth as r increases for a fixed endowment?)

To this point we have mentioned tastes (indifference maps) and opportunities (the market line) separately. Now we join the two together to show how the individual selects that most preferred consumption set from among his feasible set (MM'). In Fig. 5 we show two individuals who trade their endowments to obtain more preferred consumption combinations.

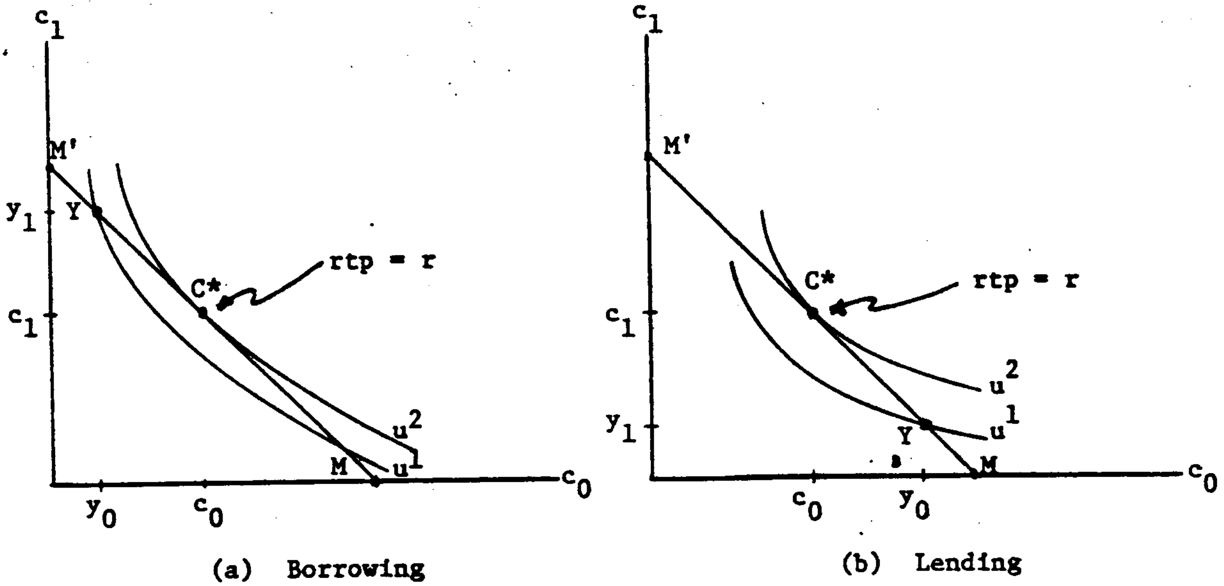


Fig. 5: Trading among Alternative Opportunities

The individual in panel (a) finds himself at Y where his MRS (slope of $u^1$) differs from the market's MRE. Here the amount of $c_1$ that he is <u>willing</u> to sacrifice for another unit of $c_0$ is greater than the market <u>requires</u> him to sacrifice, i.e., rtp > r. For example, he might have rtp = .3 which means he discounts future consumption at 30% whereas the market might require a discount on future consumption of only 10% as when r = .1 . He would find it beneficial to trade on these terms. Thus he exchanges some $c_1$ for more $c_0$ and moves down MM' to a higher indifference curve. He continues trading until he reaches C* where $u^2$ is just tangent to MM'. This is the most preferred opportunity since any additional (or less) trading results in a combination lying below $u^2$, the highest achievable utility. At C* the slope of $u^2$ and MM' are equal so MRS = MRE or rtp = r. That is, the rate at which he is willing to exchange is just equal to the rate at which he can exchange. Total borrowing is given by

$$b_0 = c_0 - y_0 > 0 .$$

The individual in (a) was relatively well endowed with $c_1$ and found borrowing would improve the utility of his consumption pattern. The one in (b) is relatively well endowed in $c_0$ so he finds lending to be optimal as he moves to C* where rtp = r and total lending is given by

$$l_0 = y_0 - c_0 > 0 .$$

Both individuals preferred consumption patterns different from their endowments. Had they been in a primitive economy without opportunities for exchange (borrowing and lending) they would have been forced to consume their endowment which is inferior to C*. But with market opportunities for exchange they moved from $u^1$ to $u^2$ . They became better off and so the difference in the utility indices $u^2$ and $u^1$ is often called the "welfare gain from exchange."

It is easy to envision how equilibrium is achieved in this Fisherian world. At the interest rate r we saw as in Fig. 5 that some agents would want to borrow and others would want to lend. If at this r, the desired borrowings exceeded the supply of lendings then r would rise. Borrowers would revise downward their borrowings and lenders increase their desired lendings. This process would continue until aggregate borrowing equalled aggregate lending. The r which brought about this equality is called the "equilibrium rate of interest." It is interesting to note that at equilibrium, rtp = r for each individual, i.e., rtp's are equal across individuals so every individual is equally impatient (or patient for that matter).

In anticipation of our next approximation we might ask whether an increase in wealth necessarily leaves a person better off. In Fig. 6 we consider for simplicity an individual whose optimal consumption just happened to coincide with his endowment. Would this individual have preferred an alternative endowment such as G or H which lie below $u^1$ and would result in lower utility if consumed without trading? The answer is yes! The endowments G and H are on a higher market line than the original Y. Thus with trade he could attain C* which is preferrable to Y.

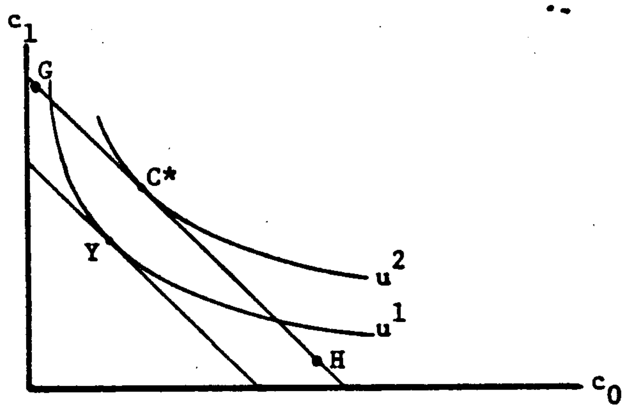


Fig. 6: Welfare Gain from More Wealth

We can see that more wealth is preferred to less wealth even though the new wealth might have a less desirable "time shape," i.e., $c_1/c_0$ ratio. This is so since the individual has perfect opportunities to transform a less desirable pattern of income into a more desirable consumption pattern.

We turn next to our second approximation in which individuals may exchange their endowment for alternative income streams through real productive investment.

## III. Second Approximation: Production and Exchange

So far the only way an individual could transform his endowment into alternative consumption patterns was by trading with other individuals with diverse preferences. Every unit of borrowing is matched by a unit of lending from the lender's endowment. In the succeeding period the borrower repays principal and interest from his endowed income. The repayment augments the lender's consumption at $t = 1$. No new consumption is created in this process as one individual's gain is another's loss. The entire community is constrained to consume no more than the sum of the endowments at any time.

Now we introduce the possibility of using part of the endowment at $t = 0$ to produce more consumption in the future. This process is called "investment." It differs from pure exchange in that something new is actually created since the supply of $c_1$ for the community as a whole is enhanced at the expense of a diminished $c_0$. For example, if the unit of consumption is corn at $t = 0$ and $t = 1$ the individual would now have three alternatives available for the disposal of $c_0$. He could eat it, lend it, or plant it to produce more corn at $t = 1$. Planting the corn would be a type of investment and the additional bushels next period would represent the returns from investment. Investment is an "exchange with nature" instead of exchange with other individuals as with borrowing and lending.

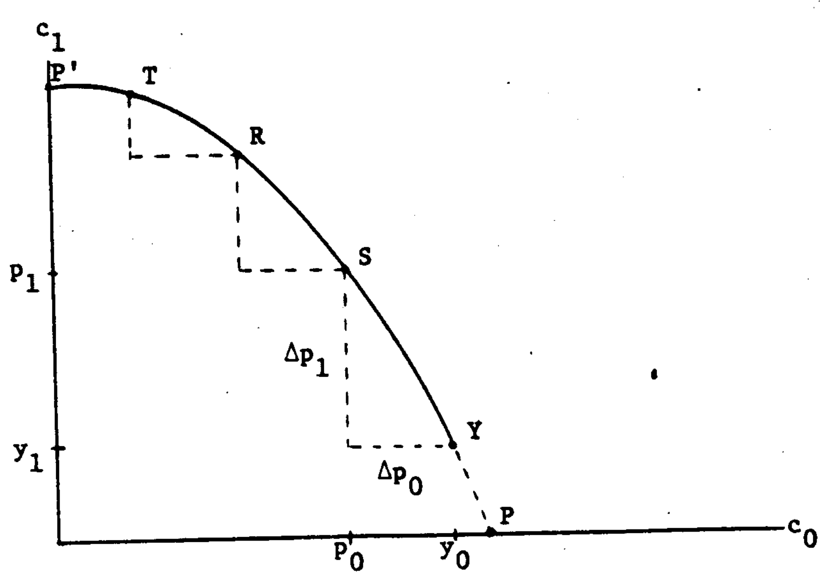


Fig. 7: Investment Opportunity Schedule

In Fig. 7 we show an individual's "investment opportunity schedule" or "production possibility curve" PP'. We will be concerned with the segment YP' which shows investment since the segment YP shows "disinvestment." PP' represents all combinations $(p_0, p_1)$ attainable by physical transformations of the endowment vector Y. This transformation schedule PP' is of the general form

$$F(p_0, p_1) = 0$$

where $p_0$ , the horizontal coordinate, is denominated in terms of $c_0$ units and $p_1$ , the vertical coordinate, in terms of $c_1$ units. The difference $q_0 = p_0 - y_0 < 0$ represents investment and $q_1 = p_1 - y_1 > 0$ represents gross returns from investment. We assume F is negatively sloped and concave to the origin.

Fig. 7 is drawn with three discrete investment opportunities S, R, and T such that the horizontal movements (sacrifices of $c_0$ for investment) $\Delta p_0$ are all equal. However, because of the concavity of F, each additional unit of investment $\Delta p_0 < 0$ produces smaller and smaller increments of return $\Delta p_1 > 0$ as we move to S, then R, and on to T. This shows "diminishing marginal returns" on investment as we move up the transformation schedule PP'. The ratio $\Delta p_1/\Delta p_0 < 0$ is analogous to the concept of the "internal rate of return" over the increment $\Delta p_0$ , denoted by $irr_\Delta$ , where

$$-(1 + irr_\Delta) = \Delta p_1/\Delta p_0$$

or

$$irr_\Delta = (\Delta p_1 + \Delta p_0)/(-\Delta p_0) \ .$$

(Recall $\Delta p_0 < 0$ because it is a sacrifice or "outflow.") For example, if a unit sacrifice of $\Delta p_0 = -1$ produced a return of $\Delta p_1 = 1.2$ units, then $irr_\Delta = .2 = 20\%$.

Another important concept is the slope at any point along PP' instead of the ratio of changes over a discrete interval. This slope which is negative is called the "marginal rate of transformation" or simply MRT. This gives rise to another rate of return notion called the "marginal rate of return" or mrr where

$$\text{MRT} = -\ (1 + \text{mrr}).$$

It will be more convenient to deal with mrr for the same reason we emphasized rtp instead of MRS. In Fig. 7, mrr declines steadily as we move away from Y since PP' is becoming less steep.

We have two measures of return so far. We spoke of $\text{irr}_\Delta$ for any discrete movement between points along YP', e.g., S to R. Then we defined mrr at any point (or for any infinitesimally small movement) along YP'. The final measure is called the "internal rate of return" or "irr" and refers to the total movement from the endowment Y to some final point like T as in Fig. 7 and reproduced in Fig. 8.

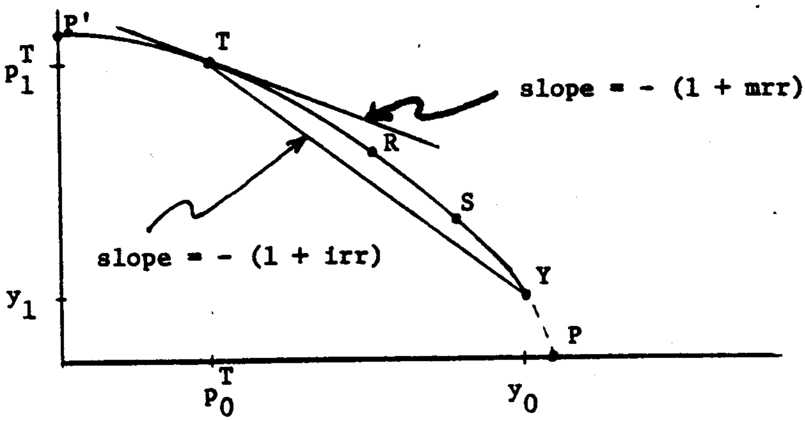


Fig. 8: irr and mrr

In moving to T, the individual gives up $y_0 - p_0^T$ in return for $p_1^T - y_1$. Thus irr would be

$$\text{irr} = [(p_1^T - y_1) - (y_0 - p_0^T)]/(y_0 - p_0^T).$$

$$\frac{\Delta \rho_1 - \Delta \rho_0}{\Delta \rho_0}$$

To make a familiar analogy, we might think of the "capital budget" as the entire investment $p_0^T - y_0$. This capital budget consists of a set of discrete movements (Y to S, S to R, and R to T) each of which has its own $irr_\Delta$. Then irr is a weighted average of the $irr_\Delta$'s on the successive increments. (This doesn't always hold for multiperiod investments.) As we move from Y to T by increments we have diminishing returns so we have

$$irr\Big|_{Y \text{ to } T} > irr_\Delta\Big|_{R \text{ to } T} > mrr\Big|_{\text{at } T}$$

(Incidentally, if an individual literally wanted to maximize his rate of return, where would he stop investing?) Note that if we treat the total movement Y to T as "one big increment" then irr and $irr_\Delta$ are the same in this case.

To this point we have introduced the concept of the transformation schedule and the individual's <u>ability</u> to transform current units into future units without mentioning his <u>willingness</u> to do so. We now unite the preference analysis of the first approximation with the investment analysis. We at first ignore borrowing and lending possibilities and consider only real transformations. For example, we might think of Robinson Crusoe marooned on a deserted island having no one with whom to borrow or lend. He can eat part of his harvest of wild corn and plant (invest) the rest to augment next period's harvest. His actions will be guided by tastes and opportunities.

We show a typical situation in Fig. 9. The individual is initially at his endowment Y on PP'. He is indifferent between point Y and point H on $u^1$. Hence he is willing to sacrifice $(h_0 - y_0)$ if he can obtain at least $(h_1 - y_1)$. Will he make the investment? Of course, since by investing along PP' to point G he can obtain $(g_1 - y_1)$ which exceeds the minimum required amount by $(h_1 - g_1)$ which is preferred to points on $u^1$.

In fact, he will attain point P* where $u^2$, the highest achievable indifference curve, is just tangent to PP'. At this equilibrium, since the slopes are equal we have MRS = MRT or rtp = mrr. Note also that he is forced to consume what he produces in the absence of borrowing and lending possibilities.

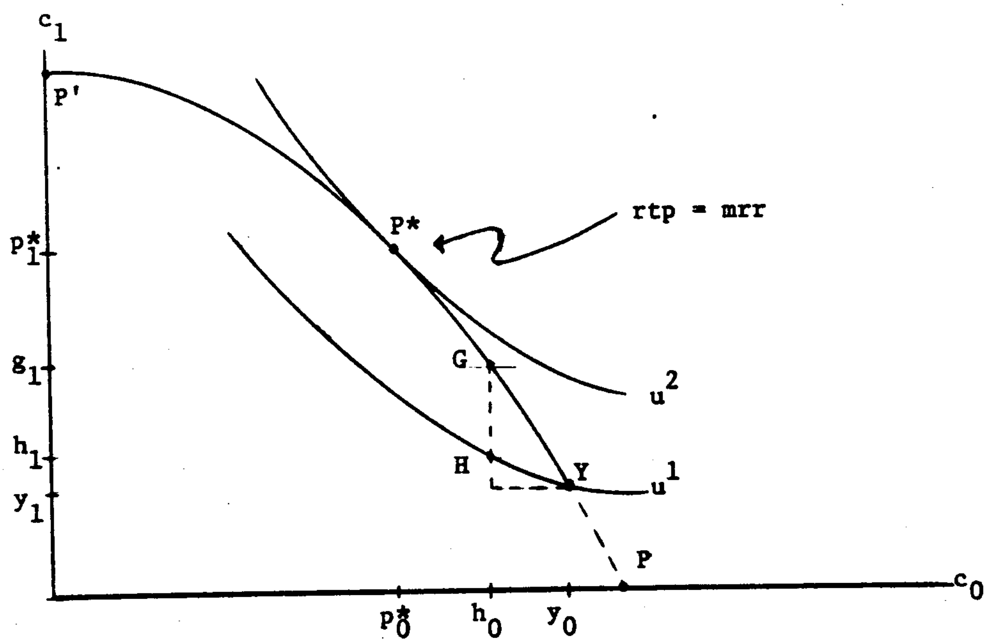


Fig. 9: Investment and Consumption without a Market

If we suppose that Robinson Crusoe had been fortunate enough to have landed on an inhabited island with a well established system of exchange (called a capital market) he would have market opportunities as shown by the market lines in Fig. 10. Having assumed ownership of his field of wild corn, he is endowed with the combination Y on $u^1$. By lending he could achieve point H on $u^2$ which is preferred to Y. A better alternative than H would be to invest along the transformation schedule to point G without borrowing and lending. Since G is on $u^3$, this is preferred to H.

The first two strategies involved only market opportunities (H) or productive opportunities (G) without using the two in conjunction. The optimal strategy involves a combination of productive and market opportunities. Should the individual move on to P* he would maximize his wealth of $W_0^P$ where the increase in wealth $W_0^P - W_0^Y$, attributable to investment, is called the "net present value" of investment. Formally,

$$W_0^P - W_0^Y = \frac{(p_1 - y_1)}{1 + r} - (y_0 - p_0)$$

where $p_1 - y_1$ is the return from investing $(y_0 - p_0)$. Of course, while P* is inferior to G from the standpoint of tastes, it is preferred because the individual may now borrow back to point C* which is on $u^4$ and is superior to G. In fact, C* is the most preferred position attainable to the individual.

Two important tangencies are shown in Fig. 10. First at point P*, MRT = MRE or equivalently mrr = r. That is, to maximize wealth the individual should continue investing so long as the marginal rate of return exceed the rate of interest, or "cost of capital," r. He stops when the two are equated, i.e., when his rate of transformation in <u>production</u> just equals his rate of transformation in market <u>exchange</u>. The second important equality lies at point C* where MRS = MRE or rtp = r. To optimize his consumption stream from wealth $W_0^P$, he continues borrowing so long as his rate of time preference exceeds the market rate. He stops when rtp = r because only at this point is the rate at which he is <u>willing</u> to exchange equal to the market rate at which he <u>can</u> exchange. Note finally, the tangencies of PP' and $u^4$ to the same market line imply the equality mrr = r = rtp. Thus the marginal rate of transformation in production equals the marginal rate of exchange in the market which in turn is equal to the marginal rate of substitution in consumption.

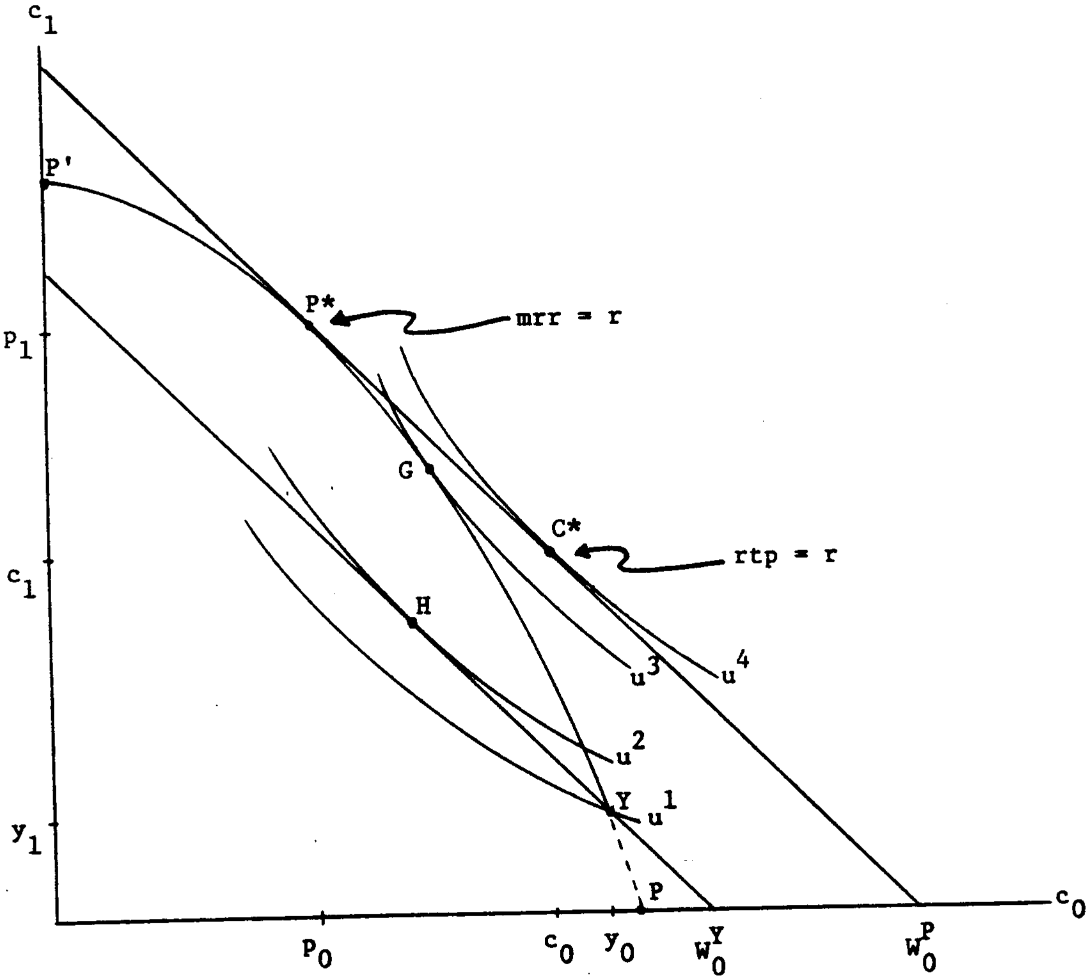


Fig. 10: Alternate Final Positions

The final equilibrium results in the maximization of wealth by investing to P* and the sequential consumption at C* maximizes utility. The reader should satisfy himself that no point along PP' other than the wealth maximizing P* will result in a higher utility.

## IV. Separation, Wealth Maximization, and Market Equilibrium

We shall summarize the findings of these notes in this final section while at the same time attempting to derive managerial implications for capital budgeting decisions and positive implications for market equilibrium. Fig. 11 summarizes the findings of Section III. We change the setting somewhat to emphasize the relationship to financial management.

Suppose an individual were to approach you with his investment-consumption problem. He informs you that he has endowment Y and investment opportunities PP'. He agrees to entrust part of his current endowment to you in much the same way that owners entrust their funds to managers for investment. He informs you that he wishes to maximize the utility of his consumption over the next two periods but he does not divulge just what his consumption preferences are.

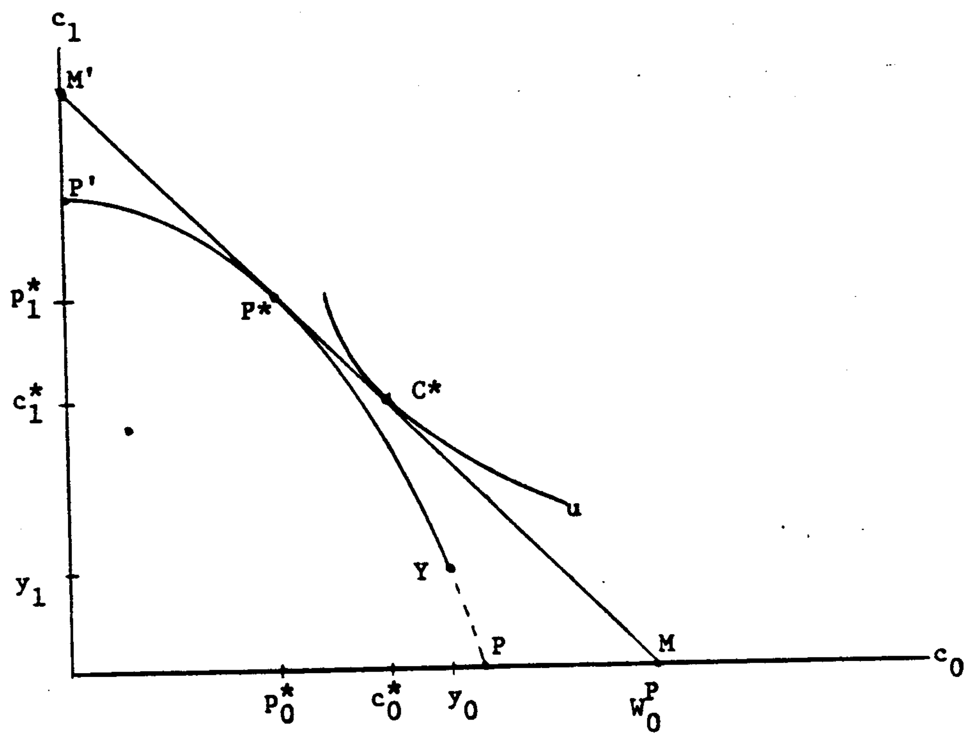


Fig. 11: Separation and Individual Equilibrium

Further, you know that the capital market is perfect in the following sense:

1. The individual can borrow or lend at the market rate r, i.e., he is a "price-taker" in the capital market;
2. There is no possibility of default on loans since the individual is constrained to borrow no more than his time one income will allow him to repay with interest;

3. There are no transactions costs; and

4. The markets are complete in that the only two commodities of interest, $c_0$ and $c_1$, are freely traded.

Can you help the individual in making his decisions without knowing his preferences? More specifically, is there some objective criterion you can use that does not depend on subjective preference (except that more is preferred to less)? Fortunately, the answer to both questions is in the affirmative. Recall from the previous section that a necessary condition to maximize utility was to achieve the highest market line first. In terms of Figure 11, the individual should invest along PP' to P* which maximizes wealth $W_0^P$. (This is equivalent to maximizing NPV as we saw before.) Further wealth is an objective measure since once the interest rate is given, any combination along PP' can be evaluated and the optimum combination selected without regard to the individual's preferences. This principle is one of the most important in all of finance and it is called the Separation Theorem. It may be stated more formally.

> Separation Theorem: In perfect and complete markets, the productive decision is to be guided only by the objective market criterion of attained wealth without regard to the individual's subjective preferences which guide the consumption decision.

Of course, once you have guided the individual to P* he must reveal his preferences before he can get to C*. However, he could not even get to C* without first moving to P* and attaining the maximum wealth $W_0^P$.

The implications of the Separation Theorem for financial managers cannot be overstated. It implies that a manager can make optimal investment decisions without any prior knowledge of individual stockholders' preferences. Suppose, for example, that when r = 10%, the manager can achieve mrr = 12% on the marginal unit of investment. If by virtue of their endowments, stockholders have, say, rtp > 15% should the manager commit more of the relatively higher valued $c_0$ to produce more of the relatively lower valued $c_1$? Yes, since investor wealth is enhanced and they would then have enhanced market opportunities to obtain whatever consumption pattern they desire. (In fact, the firm could actually do the borrowing to finance the initial investment instead of having owners contribute equity funds and then borrow on personal account.) This is especially important when owners initially have quite diverse preferences. Hence, they need not be polled to set a "collective" firm goal.

We have derived the highly desirable Separation Theorem for managerial decisions. Fisher, however, was additionally concerned with the final equilibrium interest rate r. Each individual would have tangencies like P* and C* in Figure 11 in the final equilibrium. Hence, mrr's and rtp's are equated across all individuals since there is but a single market rate r for all. As in the pure exchange model, borrowings must be matched identically by lendings. Thus, in Figure 11 the borrowing

$$b_0 = c_0^* - p_0^*$$

must be matched by a lender (or combination of lenders.)

Another familiar macroeconomic concept is the investment-savings relationship. For the individual, we have savings

$$s_0 = y_0 - c_0^* .$$

His savings are positive since he consumes less than his endowment. The final notion is investment which here is

$$i_0 = y_0 - p_0^* .$$

Just as the community as a whole cannot borrow more than it lends, it cannot invest more than it saves. Hence, aggregate savings equals aggregate investment. The equilibrium interest rate that brings about this simultaneous equality is called the market rate of interest.