## **CHAPTER 5**

**Net Present Value and Other Investment Criteria**

## **Answers to Problem Sets**

1. a. A = 3 years, B = 2 years, C = 3 years

 b. B

 c. A, B, and C

 d. B and C (NPVB = $3,378; NPV*C* = $2,405)

 e. True

 f. It will accept no negative-NPV projects but will turn down some with

 positive NPVs. A project can have positive NPV if all future cash flows are

 considered but still do not meet the stated cutoff period.

2. Given the cash flows C0, C1, . . . , C*T*, IRR is defined by:

 

 It is calculated by trial and error, by financial calculators, or by spreadsheet

 programs.

3. a. $15,750; $4,250; $0

 b. 100%.

4. No (you are effectively “borrowing” at a rate of interest higher than the opportunity cost of capital).

5. a. Two

 b. -50% and +50%

 c. Yes, NPV = +14.6.

6. The incremental flows from investing in Alpha rather than Beta are -200,000; +110,000; and 121,000. The IRR on the incremental cash flow is 10% (i.e., -200 + 110/1.10 + 121/1.102 = 0). The IRR on Beta exceeds the cost of capital and so does the IRR on the incremental investment in Alpha. Choose Alpha.

7. 1, 2, 4, and 6

8. a. 





1. Payback A = 1 year

Payback B = 2 years

Payback C = 4 years

1. A and B

d. 

The present value of the cash inflows for Project A never recovers the initial outlay for the project, which is always the case for a negative NPV project.

The present values of the cash inflows for Project B are shown in the third row of the table below, and the cumulative net present values are shown in the fourth row:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| C0 | C1 | C2 | C3 | C4 | C5 |
| -2,000.00 | +1,000.00 | +1,000.00 | +4,000.00 | +1,000.00 | +1,000.00 |
| -2,000.00 | 909.09 | 826.45 | 3,005.26 | 683.01 | 620.92 |
|  | -1,090.91 | -264.46 | 2,740.80 | 3,423.81 | 4,044.73 |

Since the cumulative NPV turns positive between year two and year three, the discounted payback period is:



The present values of the cash inflows for Project C are shown in the third row of the table below, and the cumulative net present values are shown in the fourth row:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| C0 | C1 | C2 | C3 | C4 | C5 |
| -3,000.00 | +1,000.00 | +1,000.00 | 0.00 | +1,000.00 | +1,000.00 |
| -3,000.00 | 909.09 | 826.45 | 0.00 | 683.01 | 620.92 |
|  | -2,090.91 | -1,264.46 | -1,264.46 | -581.45 | 39.47 |

Since the cumulative NPV turns positive between year four and year five, the discounted payback period is:



e. Using the discounted payback period rule with a cutoff of three years, the firm would accept only Project B.

9. a. When using the IRR rule, the firm must still compare the IRR with the opportunity cost of capital. Thus, even with the IRR method, one must specify the appropriate discount rate.

b. Risky cash flows should be discounted at a higher rate than the rate used to discount less risky cash flows. Using the payback rule is equivalent to using the NPV rule with a zero discount rate for cash flows before the payback period and an infinite discount rate for cash flows thereafter.

10.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | r =  | -17.44% | 0.00% | 10.00% | 15.00% | 20.00% | 25.00% | 45.27% |
| Year 0 |  -3,000.00 |  -3,000.00 |  -3,000.00 |  -3,000.00 |  -3,000.00 |  -3,000.00 |  -3,000.00 |  -3,000.00 |
| Year 1 |  3,500.00 |  4,239.34 |  3,500.00 |  3,181.82 |  3,043.48 |  2,916.67 |  2,800.00 |  2,409.31 |
| Year 2 |  4,000.00 |  5,868.41 |  4,000.00 |  3,305.79 |  3,024.57 |  2,777.78 |  2,560.00 |  1,895.43 |
| Year 3 |  -4,000.00 |  -7,108.06 |  -4,000.00 |  -3,005.26 |  -2,630.06 |  -2,314.81 |  -2,048.00 |  -1,304.76 |
|  | PV =  |  -0.31 |  500.00 |  482.35 |  437.99 |  379.64 |  312.00 |  -0.02 |

 The two IRRs for this project are (approximately): –17.44% and 45.27%

 Between these two discount rates, the NPV is positive.

11. a. The figure on the next page was drawn from the following points:

|  |  |
| --- | --- |
|  | Discount Rate |
|  | 0% | 10% | 20% |
| NPVA | +20.00 | +4.13 | -8.33 |
| NPVB | +40.00 | +5.18 | -18.98 |
|  |  |  |  |

1. From the graph, we can estimate the IRR of each project from the point where its line crosses the horizontal axis:

IRRA = 13.1% and IRRB = 11.9%

1. The company should accept Project A if its NPV is positive and higher than that of Project B; that is, the company should accept Project A if the discount rate is greater than 10.7% (the intersection of NPVA and NPVB on the graph below) and less than 13.1%.
2. The cash flows for (B – A) are:

C0 = $ 0

C1 = –$60

C2 = –$60

C3 = +$140

Therefore:

|  |  |
| --- | --- |
|  | Discount Rate |
|  | 0% | 10% | 20% |
| NPVB-A | +20.00 | +1.05 | -10.65 |

IRRB-A = 10.7%

The company should accept Project A if the discount rate is greater than 10.7% and less than 13.1%. As shown in the graph, for these discount rates, the IRR for the incremental investment is less than the opportunity cost of capital.

12. a. Because Project A requires a larger capital outlay, it is possible that Project A has both a lower IRR and a higher NPV than Project B. (In fact, NPVA is greater than NPVB for all discount rates less than 10 percent.) Because the goal is to maximize shareholder wealth, NPV is the correct criterion.

1. To use the IRR criterion for mutually exclusive projects, calculate the IRR for the incremental cash flows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | C0 | C1 | C2 | IRR |
| A - B | -200 | +110 | +121 | 10% |

Because the IRR for the incremental cash flows exceeds the cost of capital, the additional investment in A is worthwhile.

c. 



13. Use incremental analysis:

|  |  |  |  |
| --- | --- | --- | --- |
|  | C1 | C2 | C3 |
| Current arrangement | -250,000 |  -250,000 | +650,000 |
| Extra shift | -550,000 | +650,000 | 0 |
| Incremental flows | -300,000 | +900,000 | -650,000 |

The IRRs for the incremental flows are (approximately): 21.13% *and* 78.87%

If the cost of capital is between these rates, Titanic should work the extra shift.

14. a. 



1. Each project has a Profitability Index greater than zero, and so both are acceptable projects. In order to choose between these projects, we must use incremental analysis. For the incremental cash flows:



The increment is thus an acceptable project, and so the larger project should be accepted, i.e., accept Project E. (Note that, in this case, the better project has the lower profitability index.)

15. Using the fact that Profitability Index = (Net Present Value/Investment), we find:

|  |  |  |
| --- | --- | --- |
| Project |  | Profitability Index |
| 1 |  | 0.22 |
| 2 |  | -0.02 |
| 3 |  | 0.17 |
| 4 |  | 0.14 |
| 5 |  | 0.07 |
| 6 |  | 0.18 |
| 7 |  | 0.12 |

Thus, given the budget of $1 million, the best the company can do is to accept Projects 1, 3, 4, and 6.

If the company accepted *all* positive NPV projects, the market value (compared to the market value under the budget limitation) would increase by the NPV of Project 5 plus the NPV of Project 7: $7,000 + $48,000 = $55,000

Thus, the budget limit costs the company $55,000 in terms of its market value.

16. The IRR is the discount rate which, when applied to a project’s cash flows, yields NPV = 0. Thus, it does not represent an opportunity cost. However, if each project’s cash flows could be invested at that project’s IRR, then the NPV of each project would be zero because the IRR would then be the opportunity cost of capital for each project. The discount rate used in an NPV calculation is the opportunity cost of capital. Therefore, it is true that the NPV rule does assume that cash flows are reinvested at the opportunity cost of capital.

17.

 a.

|  |  |  |  |
| --- | --- | --- | --- |
| C0 |  = –3,000 | C0 |  = –3,000 |
| C1 |  = +3,500 | C1 |  = +3,500 |
| C2 |  = +4,000 | C2 + PV(C3 | ) = +4,000 – 3,571.43 = 428.57 |
| C3 |  = –4,000 | MIRR = 27.84% |

b. 

(1.122)(xC1) + (1.12)(xC2) = –C3

(x)[(1.122)(C1) + (1.12C2)] = –C3









Now, find MIRR using either trial and error or the IRR function (on a financial calculator or Excel). We find that MIRR = 23.53%.

It is not clear that either of these modified IRRs is at all meaningful. Rather, these calculations seem to highlight the fact that MIRR really has no economic meaning.

18. Maximize: NPV = 6,700*x*W + 9,000*x*X + 0XY – 1,500*x*Z

subject to: 10,000*x*W + 0*x*X + 10,000*x*Y + 15,000*x*Z ≤ 20,000

10,000*x*W + 20,000*x*X – 5,000*x*Y – 5,000*x*Z ≤ 20,000

0*x*W - 5,000*x*X – 5,000*x*Y – 4,000*x*Z ≤ 20,000

0 ≤ *x*W ≤ 1

0 ≤ *x*X ≤ 1

0 ≤ *x*Z ≤ 1

Using Excel Spreadsheet Add-in Linear Programming Module:

Optimized NPV = $13,450

with *x*W = 1; *x*X = 0.75; *x*Y = 1 and *x*Z = 0

If financing available at t = 0 is $21,000:

Optimized NPV = $13,500

with *x*W = 1; *x*X = (23/30); *x*Y = 1 and *x*Z = (2/30)

Here, the shadow price for the constraint at t = 0 is $50, the increase in NPV for a $1,000 increase in financing available at t = 0.

In this case, the program viewed xZ as a viable choice even though the NPV of Project Z is negative. The reason for this result is that Project Z provides a positive cash flow in periods 1 and 2.

If the financing available at t = 1 is $21,000:

Optimized NPV = $13,900

with *x*W = 1; *x*X = 0.8; *x*Y = 1 and *x*Z = 0

Hence, the shadow price of an additional $1,000 in t =1 financing is $450.