Solutions handwritten

Micro Seminar 5315

Consumer theory midterm, 2013

Instructions: Answer each question thoroughly. All answers must be legible. Please indicate your solution by circling the answer.

1. (20 points) The consumer seeks to maximize the following utility function

$$U(x,y) = -y^{-1} - x^{-1} \tag{1}$$

subject to the budget constraint $I{=}P_xX{+}P_yY$.

(a) Calculate Marshallian demand for $x(P_x, P_y, I)$. Rigorously show the effect of an increase in the price of x on demand for x.

(b) It is common to hear someone say that money cannot buy *love* or *happiness*,[1] but what about utility? Rigorously show the effect of an increase in exogenous income on indirect utility using the utility function provided above.

2. (20 points) Suppose that a consumer has the following expenditure function:

$$E(P_x, P_y, V) = P_yV + 2P_x^{\frac{1}{2}}P_y^c \tag{2}$$

(a) What is the value of the parameter, c? Explain your answer.

(b) Rewrite the expenditure function with the value of c found in part a. Use Shepherd's Lemma to derive the compensated demand for $y^c(P_x, P_y, V)$.[2] Rigorously show the effect of a change in the price of x on demand. Are these goods net complements, net substitutes, gross complements or gross substitutes?

3. (15 points) A consumer has Cobb-Douglas utility, $U = f(x,y) = (xy)^{\frac{1}{2}}$. Each good can only be purchased in markets at prices, P_x and P_y, using exogenous (non-labor) income, I. Assume that (i) $x, y > 0$; (ii) $P_x = 1, P_y = 4, I = 8$.[3] The demand functions are provided for you: $x(P_x, P_y, I) = \frac{I}{2P_x}$ and $y(P_x, P_y, I) = \frac{I}{2P_y}$. The government wishes to collect tax revenue and is choosing between a "per-unit tax" (excise tax) or a lump-sum income tax.

(a) Calculate the demand for x and y if the government taxes each unit of x at 1.

(b) Assume government taxes non-labor income, I, by an amount equalling the tax revenue collected under the excise tax from the previous question. Calculate demand for x and y under this lump-sum income tax.

(c) Which tax regime does the consumer prefer? Why?

[1] The Beatles wrote, "I don't care too much for money / For money can't buy me love".

[2] You are not required to prove Shepherd's Lemma.

[3] All prices and income are in US dollars for simplicity. There are no externalities or leisure in this problem, either.

1. Let $u(x,y) = -\frac{1}{Y} - \frac{1}{X}$

(a) $\mathcal{L} = -\frac{1}{Y} - \frac{1}{X} + \lambda(I - P_x X - P_y Y)$

FOC1: $\frac{\partial \mathcal{L}}{\partial X} = \frac{1}{X^2} - \lambda P_x = 0$

FOC2: $\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{Y^2} - \lambda P_y = 0$

$\lambda = \frac{1}{P_x X^2} = \frac{1}{P_y Y^2}$

$P_x X^2 = P_y Y^2$

$X^2 = \frac{P_y}{P_x} y^2$

FOC3: $\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x X - P_y Y = 0$

$X = \left(\frac{P_y}{P_x}\right)^{\frac{1}{2}} y$

Substitute X into FOC3.

$I - P_y Y - P_x X = 0$

$I = P_y Y + P_x \left[\left(\frac{P_y}{P_x}\right)^{1/2} y\right]$

$I = P_y Y + (P_x P_y)^{1/2} y$

$I = y(P_y + (P_x P_y)^{1/2})$

$$\boxed{y^*(P_x, P_y, I) = \frac{I}{P_y + (P_x P_y)^{1/2}}}$$

$X^*(P_x, P_y, I) = \left(\frac{P_y}{P_x}\right)^{1/2} \left(\frac{I}{P_y + (P_x P_y)^{1/2}}\right)$

$= \frac{P_y^{1/2} I}{(P_x^{1/2} P_y) + P_y^{1/2} P_x}$

$= \cancel{\left(\frac{P_y^{1/2}}{P_y^{1/2}}\right)} \frac{I}{(P_x P_y)^{1/2} + P_x}$

$$\boxed{X^*(P_x, P_y, I) = \frac{I}{P_x + (P_x P_y)^{1/2}}}$$

$$\boxed{\frac{\partial X(P_x, P_y, I)}{\partial P_x} = \frac{\overset{(-)(+)}{-I}}{\underbrace{[P_x + (P_x P_y)^{1/2}]^2}_{(+)}} \cdot \underbrace{\left[1 + \left(\frac{P_y}{4P_x}\right)^{\frac{1}{2}}\right]}_{(+)} < 0}$$

$$\boxed{\frac{\partial X}{\partial P_x} < 0}$$

1. (b) Find indirect utility.

$$U\left(x^*[P_x, P_y, I], y^*[P_x, P_y, I]\right) =$$

$$V = \frac{-1}{x^*} - \frac{1}{y^*}$$

$$= \frac{-1}{\frac{I}{P_x + (P_x P_y)^{\frac{1}{2}}}} - \frac{1}{\frac{I}{P_y + (P_x P_y)^{\frac{1}{2}}}}$$

$$V = \frac{-P_x - (P_x P_y)^{\frac{1}{2}} - P_y - (P_x P_y)^{\frac{1}{2}}}{I}$$

$$V(P_x, P_y, I) = -\frac{\left(P_x + 2(P_x P_y)^{\frac{1}{2}} + P_y\right)}{I}$$

$$\left[\frac{\partial V(P_x, P_y, I)}{\partial I} = \frac{[P_x + 2(P_x P_y)^{\frac{1}{2}} + P_y]}{I^2} > 0\right]$$

An increase in non-labor income caused an increase in indirect utility.

$$\frac{\partial V}{\partial I} > 0$$

2. $E(P_x, P_y, \bar{V}) = P_y \bar{u} + 2 P_x^{1/2} P_y^{c}$

a. expenditure functions are homogeneous of degree one (H^1) in all prices. Therefore $\boxed{c = 1/2}$ See:

$$E(tP_x, tP_y, \bar{u}) = tP_y \bar{u} + 2(tP_x)^{1/2}(tP_y)^{c}$$

$$= t^1 P_y \bar{u} + t^{\frac{1}{2}+c} P_x^{1/2} P_y^{c}$$

$=$

if $\frac{1}{2} + c = 1$, then $c = 1 - 1/2 = 1/2$.

$$= t P_y \bar{u} + t P_x^{1/2} P_y^{1/2}$$

$$= t \left(P_y \bar{u} + (P_x P_y)^{1/2} \right)$$

$$\boxed{E(tP_x, tP_y, \bar{u}) = t E(P_x, P_y, \bar{u})}$$

b. $E(P_x, P_y, \bar{u}) = P_y \bar{u} + 2(P_x P_y)^{1/2}$

Shepherd's Lemma: $\dfrac{\partial E(P_x, P_y, \bar{u})}{\partial P_x} = x^c(P_x, P_y, \bar{u})$

$\therefore \dfrac{\partial E(P_x, P_y, \bar{u})}{\partial P_x} = \cancel{\tfrac{1}{2}} \cdot 2 \, P_x^{-\frac{1}{2}} P_y^{\frac{1}{2}}$ $\qquad \dfrac{\partial E(P_x, P_y, \bar{u})}{\partial P_y} = \bar{u} + \frac{1}{2} \cdot 2 \left(\dfrac{P_x}{P_y}\right)^{1/2}$

$\boxed{x^c(P_x, P_y, \bar{u}) = \left(\dfrac{P_y}{P_x}\right)^{\frac{1}{2}}}$ and $\boxed{y^c(P_x, P_y, \bar{u}) = \bar{u} + \left(\dfrac{P_x}{P_y}\right)^{1/2}}$

$\cancel{\dfrac{\partial x^c(P_x, P_y, \bar{u})}{\partial P_x}}$ $\qquad \boxed{\dfrac{\partial y^c(P_x, P_y, \bar{u})}{\partial P_y} = -\dfrac{1}{2} \dfrac{P_x^{\frac{1}{2}}}{P_y^{3/2}} < 0}$

Net substitutes $\longrightarrow$ $\boxed{\dfrac{\partial y^c(P_x, P_y, \bar{u})}{\partial P_x} = \dfrac{1}{2(P_x P_y)^{1/2}} > 0}$

3. $u = f(x,y) = (xy)^{\frac{1}{2}}$

$P_x = 1$

$P_y = 4$

$I = 8$

$x(P_x, P_y, I) = \frac{I}{2P_x}$

$y(P_x, P_y, I) = \frac{I}{2P_y}$

Baseline (no tax).

$$\max u = V(P_x, P_y, I) = \left[\frac{I}{2P_x}\right]^{\frac{1}{2}} \left[\frac{I}{2P_y}\right]^{\frac{1}{2}}$$

$$\boxed{V(P_x, P_y, I) = \frac{I}{2(P_x P_y)^{\frac{1}{2}}}}$$

$P_x = 1$; $P_y = 4$; $I = 8$

$$V(1,4,8) = \frac{8}{2(1 \cdot 4)^{1/2}} = \frac{4}{2} = 2 \text{ utils}$$

$$\boxed{V(1,4,8) = 2 \text{ utils}} \quad (\text{no tax})$$

(a) Excise tax ($t = 1$ per unit of x consumed)

$P_x + t = \tilde{P}_x$

$\tilde{P}_x = 2$

$P_y = 4$

$I = 8$

$$x(\tilde{P}_x, P_y, I) = \frac{I}{2\tilde{P}_x} = \frac{8}{2 \cdot 2} = 2 \text{ units of } x$$

$$y(\tilde{P}_x, P_y, I) = \frac{I}{2P_y} = \frac{8}{2 \cdot 4} = 1 \text{ unit of } y$$

$$V(\tilde{P}_x, P_y, I) = \frac{I}{2(\tilde{P}_x P_y)^{\frac{1}{2}}} = \frac{8}{2(2 \cdot 4)^{\frac{1}{2}}} = \frac{4}{2 \cdot 2^{1/2}} = \frac{2}{2^{1/2}}$$

$2^{1/2} = 1.4$ $\quad V = 2 \div 1.4 = 1.43$

$$\boxed{V(\tilde{P}_x, P_y, I) = 1.43 \text{ utils}}$$

3. (b) Lump-Sum tax

First find tax revenue collected in part 3a.

tax revenue (TR) $= t \cdot X(\tilde{P_x}, P_y, I)$

$= \$1 \cdot 2$ units demanded

$\boxed{TR = \$2 \text{ in total tax revenue}}$

Second, set tax on income at $TR = \$2$. Therefore, $\tilde{I} = I - TR = 8 - 2 = \$6$. P_x + P_y are 1 and 4.

$\tilde{I} = 6$

$P_x = 1$

$P_y = 4$

$$\boxed{X(P_x, P_y, \tilde{I}) = \frac{\tilde{I}}{2P_x} = \frac{6}{2 \cdot 1} = 3 \text{ units of } X}$$

$$\boxed{Y(P_x, P_y, \tilde{I}) = \frac{\tilde{I}}{2P_y} = \frac{6}{2 \cdot 4} = \frac{6}{8} = \frac{3}{4} \text{ unit of } Y}$$

$$V(P_x, P_y, \tilde{I}) = \frac{\tilde{I}}{2(P_x P_y)^{\frac{1}{2}}}$$

$$= \frac{6}{2(1 \cdot 4)^{\frac{1}{2}}}$$

$$= \frac{3}{2}$$

$$\boxed{V(P_x, P_y, \tilde{I}) = 1.5 \text{ utils}}$$

3. (c).

tax regime	$V(P_x, P_y, I)$	Demand x	Demand y
baseline	2 utils	4 units	1 unit
excise tax	1.43 utils	2 units	1 unit
lumpsum	1.5 utils	3 units	3/4 units

Consumer prefers lump sum tax regime to excise tax regime b/c lump sum indirect utility is larger than excise tax $(1.5 > 1.43)$.

4. Only equations ④ + ⑤ are demand equations. Marshallian demand equations are homogeneous of degree zero (H^0) in all prices and income. Only ④ + ⑤ are H^0 in all prices + income.

X ③ $y(tP_x, tP_y, tI) = \dfrac{tI}{2tP_x tP_y} = \dfrac{tI}{t^2 2P_xP_y} = \left(\dfrac{1}{2}\right)\dfrac{I}{2P_xP_y}$

$= \dfrac{1}{2}\, y(P_x, P_y, I)$

✓ ④ $y(tP_x, tP_y, tI) = \dfrac{tI}{(tP_x tP_y)^{1/2} + tP_y} = \dfrac{tI}{t(P_xP_y)^{1/2} + tP_y}$

$= \dfrac{\cancel{t}(I)}{\cancel{t}\left[(P_xP_y)^{1/2} + P_y\right]}$

$= \dfrac{I}{(P_xP_y)^{1/2} + P_y}$

$= y(P_x, P_y, I)$

✓ ⑤ $y(tP_x, tP_y, tI) = \dfrac{tI + tP_y - tP_x}{2tP_x}$

$= \dfrac{\cancel{t}(I + P_y - P_x)}{\cancel{t}\; 2P_x}$

$= y(P_x, P_y, I)$

5. (a). In a model with two goods, x and y, if x is inferior, then x and y are gross complements in response to an increase in the price of y. $\boxed{\text{FALSE}}$

$$X(P_x, P_y, E(P_x, P_y, \bar{u})) = X^c(P_x, P_y, \bar{u})$$

$$\frac{\partial X}{\partial P_y} + \frac{\partial X}{\partial E}\frac{\partial E}{\partial P_y} = \frac{\partial X^c}{\partial P_y}$$

$$\frac{\partial X}{\partial P_y} = \frac{\partial X^c}{\partial P_y} - \frac{\partial X}{\partial E}\frac{\partial E}{\partial P_y}$$

$$\frac{\partial X}{\partial P_y} = \frac{\partial X^c}{\partial P_y} - y^c(P_x, P_y, \bar{u})\frac{\partial X}{\partial I}$$

If we assume continuous, transitive and complete preferences, then x will have diminishing MRS under quasi-concave utility functions. As a result, indifference curves in 2-dimensional space are convex, which means that $\frac{\partial X^c}{\partial P_y} > 0$.

This is simply a corollary of Hicks' 1st Law. The demand function, $y^c > 0$, and we are given $\frac{\partial X}{\partial I} < 0$. Therefore,

we can sign the ~~Slutsky~~ partial derivative, $\frac{\partial X}{\partial P_y}$, using the Slutsky equation:

$$\frac{\partial X}{\partial P_y} = (+) + (-)(+)(-)$$
$$= (+) + (+) > 0$$

Or, $\boxed{\frac{\partial X}{\partial P_y} > 0}$. If $\partial X/\partial P_y > 0$, X and y are gross substitutes.

5b) According to Hicks, most goods in the economy are substitutes. $\boxed{\text{TRUE}}$

Use the compensated demand function with n-goods and apply Euler's theorem. As compensated demand is homogeneous of degree zero in all prices, this is:

$$X_1^c(P_1, P_2, \dots, P_n, \bar{u})$$

$$\frac{\partial X_1^c}{\partial P_1} P_1 + \frac{\partial X_1^c}{\partial P_2} P_2 + \dots + \frac{\partial X_1^c}{\partial P_n} P_n = 0 \qquad (\text{Euler})$$

$$\frac{\partial X_1^c}{\partial P_1} \cdot \frac{P_1}{X_1^c} + \frac{\partial X_1^c}{\partial P_2} \cdot \frac{P_2}{X_1^c} + \dots + \frac{\partial X_1^c}{\partial P_n} \cdot \frac{P_n}{X_1^c} = 0 \quad (\text{Divide by } X_1^c)$$

$$e^c_{1,P1} + e^c_{1,P2} + \dots + e^c_{1,PN} = 0 \qquad (\text{elasticity defn})$$

Rewrite $e^c_{1,P2} + \dots + e^c_{1,PN}$ as $\sum_{i=2}^{N} e_{1,Pi}$

$$e^c_{1,P1} + \sum_{i=2}^{N} e^c_{1,Pi} = 0$$

If $e^c_{1,P1} \leq 0$, then homogeneity of compensated demand implies $\sum_{i=2}^{N} e^c_{1,Pi} \geq 0$, which means "most goods are substitutes".

6. If $\boxed{\frac{\partial x_1}{\partial P_1}} > 0$ (Giffen), then according to Slutsky:

$$\frac{\partial x_1}{\partial P_1} = \frac{\partial X_1^c}{\partial P_1} - X_1^c(P_1, P_2, \bar{u})\frac{\partial X_1}{\partial I} > 0$$

① $\frac{\partial X_1^c}{\partial P_1} > \frac{\partial X_1}{\partial I} \cdot X_1^c(P_1, P_2, \bar{u})$;

② $\boxed{\frac{\partial X_1}{\partial I} < 0}$

Show that $\frac{\partial X_2}{\partial P_1}$ and $\frac{\partial X_2^c}{\partial P_1}$ are of opposing signs.

(i) $\frac{\partial X_2^c}{\partial P_1} > 0$

Compensated demand is H^0 in all prices. Euler's theorem:

$$X_2^c(P_1, P_2, \bar{u})$$

$$\frac{\partial X_2^c}{\partial P_1} P_1 + \frac{\partial X_2^c}{\partial P_2} P_2 = 0$$

$$\frac{\partial X_2^c}{\partial P_1} \cdot \frac{P_1}{X_2^c} + \frac{\partial X_2^c}{\partial P_2} \cdot \frac{P_2}{X_2^c} = 0$$

$$e^c_{2,P_1} + e^c_{2,P_2} = 0$$

$$e^c_{2,P_1} = -e^c_{2,P_2}$$

By assumption, $e^c_{2,P_2} < 0$ due to curvature of indiff. curve. Therefore $-e^c_{2,P_2} > 0$ which requires $\boxed{e^c_{2,P_1} > 0}$. This therefore means that $\boxed{\frac{\partial X_2^c}{\partial P_1} > 0.}$

(ii) $\frac{\partial X_2}{\partial P_1} < 0$

If $\frac{\partial X_1}{\partial P_1} > 0$, then $\frac{\partial X_1}{\partial I} < 0$ ("inferior"). Use Cournot aggregation to examine the sign of $\frac{\partial X_2}{\partial P_1}$. Diff. budget constraint (using demand functions, $X_1(P_1, P_2, I)$ and $X_2(P_1, P_2, I)$) with respect to P_1.

6. (cont.)

$$P_1 X_1(P_1, P_2, I) + P_2 X_2(P_1, P_2, I) = I$$

diff. wrt P_1.

$$P_1 \frac{\partial X_1}{\partial P_1} + X_1 + P_2 \frac{\partial X_2}{\partial P_1} = 0$$

Convert to Cournot elasticity expression.

$$\Delta_1 e_{1,P_1} + \Delta_1 + \Delta_2 e_{2,P_1} = 0$$

Rewrite as e_{2,P_1}:

$$\Delta_2 e_{2,P_1} = -(\Delta_1 + \Delta_1 e_{1,P_1})$$

$$\Delta_2 e_{2,P_1} = -\Delta_1 (1 + e_{1,P_1})$$

$$e_{2,P_1} = -\frac{\Delta_1}{\Delta_2}\left[1 + e_{1,P_1}\right]$$

If $e_{1,P_1} > 0$ b/c $\frac{\partial X_1}{\partial P_1} > 0$ (by assumption) then

the interior term, $1 + e_{1,P_1}$, is positive. Both Δ_1 + Δ_2 are positive. Therefore the negative, $-\frac{\Delta_1}{\Delta_2}$, determines the sign of e_{2,P_1}. $\boxed{e_{2,P_1} < 0.}$

∴ If $\frac{\partial X_1}{\partial P_1} > 0$ (Giffen), then using the homogeneity of degree zero of compensated demand functions, we showed this implied $\boxed{\frac{\partial X_2^c}{\partial P_1} > 0}$. We then used the Cournot aggregation to

show that if $\frac{\partial X_1}{\partial P_1} > 0$, then $e_{1,P_1} > 0$, and

therefore $e_{2,P_1} < 0$. If $e_{2,P_1} < 0$, then $\boxed{\frac{\partial X_2}{\partial P_1} < 0.}$

And therefore we showed that $\frac{\partial X_2^c}{\partial P_1}$ and $\frac{\partial X_2}{\partial P_1}$ are

of opposite signs.

BONUS QUESTION #1 (Shephard's Lemma)

$E(P_x, P_y, \bar{u})$ (expenditure fn.)

$X^c(P_x, P_y, \bar{u}) = \dfrac{\partial E(P_x, P_y, \bar{u})}{\partial P_x}$ (Shephard's Lemma).

<u>Proof</u>: $E = \min \mathcal{L} = \min \; P_x X^c(P_x, P_y, \bar{u}) + P_y y^c(P_x, P_y, \bar{u})$
$+ \lambda(P_x, P_y, \bar{u})(\bar{u} - u[x^c(P_x, P_y, \bar{u}), y^c(P_x, P_y, \bar{u})])$

$$\frac{\partial E}{\partial P_x} = \frac{\partial \mathcal{L}}{\partial P_x} = X^c(P_x, P_y, \bar{u}) + P_x \frac{\partial X^c}{\partial P_x} + P_y \frac{\partial y^c}{\partial P_x} + \frac{\partial \lambda}{\partial P_x}(\text{b.c.}) - \lambda \frac{\partial u}{\partial X^c}\frac{\partial X^c}{\partial P_x} - \lambda \frac{\partial u}{\partial y^c}\frac{\partial y^c}{\partial P_x}$$

$\downarrow$

Recall for E, the following hold:

1. $\bar{u} - u(x,y) = 0$ (FOC3)
2. $P_x - \lambda \frac{\partial u}{\partial x} = 0$ (FOC1)
3. $P_y - \lambda \frac{\partial u}{\partial y} = 0$ (FOC2)

$$\frac{\partial E}{\partial P_x} = X^c(P_x, P_y, \bar{u}) + P_x \frac{\partial X^c}{\partial P_x} - \lambda \frac{\partial u}{\partial X^c}\frac{\partial X^c}{\partial P_x} + P_y \frac{\partial y^c}{\partial P_x} - \lambda \frac{\partial u}{\partial y^c}\frac{\partial y^c}{\partial P_x} + \cancel{\frac{\partial \lambda}{\partial P_x}(\bar{u} - u(x,y))}$$

$\uparrow$ FOC3 $= 0$

$$= X^c(P_x, P_y, \bar{u}) + \underbrace{\cancel{\left(P_x - \lambda \frac{\partial u}{\partial X^c}\right)}}_{\text{FOC1}=0}\frac{\partial X^c}{\partial P_x} + \underbrace{\cancel{\left(P_y - \lambda \frac{\partial u}{\partial y^c}\right)}}_{\text{FOC2}=0}\frac{\partial y^c}{\partial P_x}$$

$$\frac{\partial E}{\partial P_x} = X^c(P_x, P_y, \bar{u})$$

Bonus #1 (cont.)

Expenditure is Concave in Prices means:

$$\frac{\partial^2 E}{\partial P_x^2} \le 0.$$

$$\frac{\partial^2 E}{\partial P_x^2} = \left(\frac{\partial}{\partial P_x}\right)\left(\frac{\partial E}{\partial P_x}\right) = \frac{\partial}{\partial P_x} x^c(P_x, P_y, \bar{u}) = \boxed{\frac{\partial x^c}{\partial P_x} \le 0 \quad \text{Hicks 1st Law}}$$

Bonus #2

$$V(P_x, P_y, I) = \max U[X(P_x,P_y,I), Y(P_x,P_y,I)] + \lambda(P_x,P_y,I)(I - P_x X(P_x,P_y,I) - P_y Y(P_x,P_y,I))$$

$$\frac{\partial V}{\partial I} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial I} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial I} + \frac{\partial \lambda}{\partial I}(b.c.) + \lambda - \lambda P_x \frac{\partial x}{\partial I} - \lambda P_y \frac{\partial y}{\partial I}$$

Recall: FOC1: $\frac{du}{dx} - \lambda P_x = 0$

FOC2: $\frac{\partial u}{\partial y} - \lambda P_y = 0$

FOC3: $I - P_x X - P_y Y = 0$

$$\frac{\partial V}{\partial I} = \frac{\partial u}{\partial x}\frac{\partial X}{\partial I} - \lambda P_x \frac{\partial X}{\partial I} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial I} - \lambda P_y \frac{\partial y}{\partial I} + \frac{\partial \lambda}{\partial I}(I - P_x X - P_y Y) + \lambda(P_x, P_y, I)$$

$$= \underbrace{\cancel{\left(\frac{\partial u}{\partial x} - \lambda P_x\right)}}_{FOC1=0}\frac{\partial X}{\partial I} + \underbrace{\cancel{\left(\frac{\partial u}{\partial y} - \lambda P_y\right)}}_{FOC2=0}\frac{\partial y}{\partial I} + \cancel{\frac{\partial \lambda}{\partial I}\underbrace{(I - P_x X - P_y Y)}_{FOC3=0}} + \lambda(P_x, P_y, I)$$

$$\frac{\partial V}{\partial I} = \lambda(P_x, P_y, I) \geq 0$$

- income will increase utility so long as consumers are consuming non-zero amounts of X & y. Which is to say $\lambda > 0$ if the budget constraint is binding. If it is not binding, then $\frac{\partial V}{\partial I}$ may be zero.