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A mathematical example of the two-echelon inventory model with asymmetric market information

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Abstract

This paper examines the extended two-echelon newsboy problem for a manufacturer and retailer in a supply chain with asymmetric market information. In this scenario, the retailer has better knowledge of the market's demand, and the manufacturer perceives the same demand. This applied model presents how the manufacturer can take advantage of the retailer's market information and improve the efficiencies in the supply chain channel.

Keywords: Single-period "newsboy" problem; Supply chain management; Inventory model; Two-echelon problem; Market-knowledge

1. Introduction

Interest in this classical single-period problem (SSP), also known as the "newsboy problem" has increased over the past several years. This interest may be characteristic in part because of the recent trend in supply chain management (SCM) and the decrease in product life cycles brought about by technological advances.

The concept of supply chain is not new to operations management. Forrester [5] along with Buffa and Miller [3] have been noted for their academic

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introduction to understanding the dynamics of scheduling systems as a whole and the impact of the activities in the production-inventory-distribution system. However, during the last decade, supply chain management has become the prevalent approach to improving logistics operations. The studies of Sterman [17], Braithwaite and Christopher [2], Lee and Billington [12], Lee et al. [13], Metters [14], and Reyes [16] coupled with the operations management's trend toward global integration, has increased the exposure of logistics and managing the supply chain. These studies not only have presented distorted demand information inefficiencies within the supply chain, but also offer ways to counteract these distortions. It is further suggested that even small improvements within the supply chain operations may result in drastic cost reductions—and therefore improving logistics operations makes practical sense.

The SSP is to find the optimal order quantity that maximizes the expected profit for a single-period probabilistic demand. The model assumes that (i) if any inventory remains after the selling period, then the item is either disposed or sold at a discount, (ii) if the order quantity is smaller than the realized demand, then the retailer forgoes some profits. The SSP is reflective of many real-life situations both in manufacturing and retailing [6], and can be used for managing capacity [18].

2. Related literature review

Lau [11] contributed an educational note for the "newsboy" single-period problem discussing the two major approaches of the "formula approach" and the "payoff matrix approach". The more popular approach taught is the "formula approach" focusing on the optimal service level and hence the optimal inventory/order quantity. Additional contributions were presented [8] for the "two-echelon" newsboy problem where the retailer has better market information than the manufacturer. They presented several decision models that should be useful for guiding a manufacturer's decisions. In addition, it was discovered that the improved retailer's market information always benefited the manufacturer (and the system), but not necessarily the retailer.

Lau and Lau [10] presented formulations and solutions for a multi-product newsboy problem with one or more capacity constraints. Pasternack [15] later considered a single inventory problem in which the vendor (retailer) has limited funds to purchase items to sell. It was assumed that the manufacturer would either sell the items to the vendor right-out or offer to the items on a consignment basis (revenue sharing) basis. This could lead to the retailer holding the inventories but on consignment and actually owned by the manufacturer.

Emmons and Gilbert [4] examined the effect of returns policies have on retailer's and manufacturer's profits when the pricing and inventory decisions occur prior to the selling season. While considering the decision variable sets of $\{(C,V),(Q,R)\}$ (see Appendix A) and concluded that when the manufacturer's start giving a retailer privileges for returning unsold goods for credit, it actually hurts the retailer. Lau and Lau [9] later showed how a monopolistic manufacturer of a single-period commodity could establish his pricing and return policies based on demand uncertainty.

Axsater [1] presented a cost structure framework that could be used for decentralized control of a multi-echelon inventory system. In this cost structure, the central warehouse pays a penalty cost in addition to the local costs (i.e. holding costs and material handling costs) for delay at the warehouse to the retailer facing the delay. This is critical for the manufacturer because they stock the single-period inventory in a warehouse prior to the actual shipment to the retailer.

For further review of the literature, it is suggested that the reader refer to Khouja [7], where a taxonomy of the single-period period literature and classified the extensions into 11-categories is presented in more detail.

3. Background information

The summary of background information [11] and [8] presented provides necessary background on the single-echelon newsboy problem and the basic two-echelon system with asymmetric market information. The definitions of symbols are given in Appendix A.

Many practical inventory problems exist where the demand for the products involved have a one-time event and are not available for subsequent selling periods. Only one order can be placed for these products to meet demand, but the demand level cannot be estimated with certainty. So the classical problem is to determine how large the single order should be.

The classical single-echelon newsboy problem approach is to find the optimal service level where if the optimal inventory/order quantity Q^* is ordered, then the optimal profit can be obtained. Hence, the classical single-echelon newsboy problem can be stated as follows: given D, M, R, and V_e, find Q^* such that

$$SL^* = \frac{(R-M)}{(R-V_e)}, \tag{1}$$

where

$$Q^* = F_D^{-1}(SL^*). \tag{2}$$

The optimal profit can then be determined by

$$\Pi = (R - V_e) \times E_D Q^*, \tag{3}$$

where for any random variable x, $E_x(q)$ is x's partial expectation with upper limit q.

$$E_x(q) = \int_0^q x g(x)\,\mathrm{d}x, \tag{4}$$

where $g(x)$ is x's density function.

The basic two-echelon system with asymmetric market information considered by Emmons and Gilbert [4], and Lau and Lau [8,9] is stated as given any C imposed by the manufacturer, the retailer will order Q^*, which is determined by (1) and (2).

$$Q^* = F^{-1}\left[\frac{(R - C)}{(R - V_e)}\right]. \tag{5}$$

The manufacturer's profits results as

$$\Pi_m = (C - M) \times Q^*. \tag{6}$$

Then by combining (5) and (6), the manufacturer's optimization problem can stated as:

Find C^* that maximizes

$$\Pi_m = (C - M) \times F^{-1}\left[\frac{(R - C)}{(R - V_e)}\right]. \tag{7}$$

Remark 1. If the manufacturer raises C in order to increase the margin component $(C - M)$ of Π_m in (6), then this higher C will also reduce the component part (5) of (7). This results in a compromise between these two opposite forces.

Remark 2. The manufacturer knows that by lowering C will not encourage a higher Q for the retailer.

After the manufacturer sets C^*, then by substituting C^* into (5), we can determine the retailer's ordering decision Q^*. Then by (3) and (4) gives the retailer's expected profits as

$$\Pi_r^* = (R - V_e) \times E_D Q^*. \tag{8}$$

The models reviewed assume symmetric market information, where the manufacturer and the retailer perceive the same demand (D).

4. Model

Lau and Lau [8] extended the two-echelon newsboy problem by studying situations in which the retailer has better market information than the manufacturer. They presented several decision models for guiding a manufacturer's decision to make pricing decisions for single-period goods. In this paper, model 1 with discrete market states is examined. In this scenario, "the retailer has better knowledge of the average demand than the manufacturer, and the manufacturer knows the extent of the retailer's superiority".

It is assumed that the market can be in one of two discrete states ($j = 1$ or 2), where in either state, the demand D_j is uniformly distributed with cumulative density function F_{D_j} with mean μ_j. The standard deviation σ is also assumed to be the same for both states. The probability that the market is in state j is P_j and therefore the overall demand D has a mean of

$$\mu_D = \mu_1 P_1 + \mu_2 P_2. \tag{9}$$

The overall demand's uncertainty has two components: σ, which is the uncertainty within each market state and the uncertainty of which of two discrete states (state 1 or 2) applies. The later is quantified as

$$\sigma_s^2 = E(\mu_i^2) - [E(\mu_i)]^2 = (\mu_1^2 P_1 + \mu_2^2 P_2) - \mu_D^2, \tag{10}$$

where the demand's uncertainties is related as

$$\sigma_D^2 = \sigma^2 + \sigma_S^2. \tag{11}$$

In this model, it is assumed that both the manufacturer and the retailer know all of the preceding market properties. Moreover, it is further assumed that the retailer knows which of the two discrete states ($j = 1$ or 2) the market is actually in; and the manufacturer does not. However, the manufacturer knows that the retailer knows and understands that he will be exposed to demand uncertainty σ_D if he does not work with the retailer. It can be further said that the retailer is also exposed to uncertainty, but a smaller σ. We can then consider the (σ_S/σ)-ratio as the market-knowledge for the retailer over the manufacturer. Hence, a high ratio would mean that the retailer has greater market-knowledge superiority over the manufacturer.

The manufacturer's optimization problem can be stated as:Find C^* that maximizes

$$\Pi_m = (C - M) \times \left(P_1 \times F_{D_1}^{-1}\left[\frac{(R-C)}{(R-V_e)}\right] + P_2 \times F_{D_2}^{-1}\left[\frac{(R-C)}{(R-V_e)}\right] \right), \tag{12a}$$

where

$$C^* = \frac{(R + V_e + 2M)}{4} + \frac{(R - V_e)\mu_D}{4\sqrt{3}\sigma}. \tag{12b}$$

Once the manufacturer sets the unit wholesale price C^*, the retailer's expected profits is:

$$\Pi_r^* = \sum_{j=1}^{2} P_j[(R - V_e) \times (E_{D_j} Q_j^*)], \tag{13}$$

where the retailer's order decision is

$$Q_j^* = F_{D_j}^{-1}\left[\frac{(R - C^*)}{(R - V_e)}\right], \quad j = 1 \text{ or } 2. \tag{14}$$

5. Numerical example

Suppose we have the retailer's estimated demand for units with the following distribution:

Sales units	Probability	Cumulative probability
500	0.1	0.1
750	0.2	0.3
1000	0.3	0.6
1250	0.2	0.8
1500	0.1	0.9
1750	0.1	1.0

Consider the parameter values: $M = 0.1$, $R = 2$, $V = V_e = 0$, $\mu_S = \mu_D = 1000$, $\sigma_D = 30$, and $\sigma_D^2 = (30)^2 = 900$. Suppose that the manufacture sets the unit wholesale price (C) at 0.5.Then by substituting into (5) gives the retailer's order decision

$$Q^* = F^{-1}\left[\frac{(R - C)}{(R - V_e)}\right] = 1500\left[\frac{(2 - 0.5)}{(2 - 0)}\right] = 1125.$$

The manufacturer's and the retailer's profits can be computed by (6) and (8), respectively.

$$\Pi_m = (C - M) \times Q^* = (0.5 - 0.1) \times 1125 = 450,$$

$$\Pi_r^* = (R - V_e) \times E_D Q^* = (2 - 0) \times 1125 = 2250$$

and are based on an assumed extreme situation of deterministic D_j—that is the retailer has perfect information. However, the market can be in one of two states ($j = 1$ or 2): increasing or decreasing. By substituting into (12) and (13), we get

$$\Pi_m = (0.5 - 0.1) \times \left(0.6 \times 1250\left[\frac{(2-0.5)}{(2-0)}\right] + 0.4 \times 1750\left[\frac{(2-0.5)}{(2-0)}\right]\right)$$
$$= 435,$$

$$\Pi_r^* = 0.6[(2-0) \times 1250] + 0.4[(2-0) \times 1750] = 2900.$$

6. Managerial implications and concluding remarks

Managing inventory in the supply chain can be modeled using the classical single-period problem. The optimization model presented in this paper is an example in its usefulness for guiding manufacturers for making pricing decisions. While it has historically been used for fashion and single-period goods, it can be extended to similar decisions for new product introduction and their respective life cycles. The manufacture will benefit more if they can take advantage of the retailer's market knowledge. Otherwise, they will do worse. Moreover, if the manufacture does take advantage of the retailer's market knowledge, and then the efficiency in the supply chain can be improved by reducing the inventory levels in the supply chain channel.

In this paper, it is assumed that the single-period model is a "one-shot" decision. However, in the market with new products and unknown life cycles, these models can be studied as continuous models. It can be suggested that further work be conducted in the area of repeated inventory decisions based on market reaction to new product introduction and their related life cycles.

Appendix A

D The inventory item's stochastic demand.

$f_D(\cdot)$, $F_D(\cdot)$ and $F_D^{-1}(\cdot)$ D's density function, cumulative density function (cfd), and inverse cfd, respectfully.

μ_D, σ_D D's mean and standard deviation respectfully.

M, V_e Product's unit manufacturing cost, and the unit salvage value obtained by the retailer from the open market after the selling season, respectfully. It is assumed $M > V_e$.

C, V Unit wholesale price that the manufacturer charges the retailer, and credit per unit paid by the manufacturer to the retailer for unsold units returned by the retailer, respectively.

Q, R Retailer's order quantity, and unit retail price that the retailer charges his customers.

Π_m, Π_r Expected profit of manufacturer and retailer, respectfully.

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