## **CHAPTER 8**

#### Portfolio Theory and the Capital Asset Pricing Model

## **Answers to Problem Sets**

1. a. 7%

b. 27% with perfect positive correlation; 1% with perfect negative correlation;

19.1% with no correlation

c. See Figure 1 below



d. No, measure risk by beta, not by standard deviation.

2. a. Portfolio A (higher expected return, same risk)

b. Cannot say (depends on investor’s attitude -toward risk)

c. Portfolio F (lower risk, same -expected return).

3. Sharpe ratio = 7.1/20.2 = .351

4. a. Figure 8.13b: Diversification reduces risk (e.g., a mixture of portfolios A

and B would have less risk than the average of A and B).

b. Those along line AB in Figure 9.13a.

1. See Figure 2 below



5. a. See Figure 3 below



b. A, D, G

c. F

d. 15% in C

e. Put 25/32 of your money in F and lend 7/32 at 12%: Expected return = 7/32

X 12 + 25/32 X 18 = 16.7%; standard deviation = 7/32 X 0 + (25/32) X 32 = 25%. If you could borrow without limit, you would achieve as high an expected return as you’d like, with correspondingly high risk, of course.

6. a. 4 + (1.41 X 6) = 12.5%

b. Amazon: 4 + (2.16 X 6) = 17.0%

c. Campbell Soup: 4 + (.30 X 6) = 5.8%

d. Lower. If interest rate is 4%, r = 4 + (1.75 X 6) = 14.5%; if rate = 6%, r = 6 +

(1.75 X 4) = 13.0%

e. Higher. If interest rate is 4%, r = 4 + (.55 X 6) = 7.3%; if rate = 6%, r = 6 +

(.55 X 4) 5 8.2%

7. a. True

b. False (it offers twice the market risk premium)

c. False

8. a. 7%

b. 7 + 1(5) + 1(-1) + 1(2) = 13%

c. 7 + 0(5) + 2(-1) + 0(2) = 5%

d. 7 + 1(5) + (-1.5)(-1) + 1(2) = 15.5%.

9. a. False – investors demand higher expected rates of return on stocks with more nondiversifiable risk.

1. False – a security with a beta of zero will offer the risk-free rate of return.
2. False – the beta will be: (1/3 × 0) + (2/3 × 1) = 0.67
3. True
4. True

10. In the following solution, security one is Campbell Soup and security two is Boeing. Then:

r1 = 0.031 σ1 = 0.158

r2 = 0.095 σ2 = 0.237

Further, we know that for a two-security portfolio:

rp = x1r1 + x2r2

σp2 = x12σ12 + 2x1x2σ1σ2ρ12 + x22σ22

Therefore, we have the following results:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| x1 | x2 | rp | σπ | σπ |
| when  = 0 | when  = 0.5 |
| 1 | 0 | 3.10% | 0.02496 | 0.02496 |
| 0.9 | 0.1 | 3.74% | 0.02078 | 0.02415 |
| 0.8 | 0.2 | 4.38% | 0.01822 | 0.02422 |
| 0.7 | 0.3 | 5.02% | 0.01729 | 0.02515 |
| 0.6 | 0.4 | 5.66% | 0.01797 | 0.02696 |
| 0.5 | 0.5 | 6.30% | 0.02028 | 0.02964 |
| 0.4 | 0.6 | 6.94% | 0.02422 | 0.03320 |
| 0.3 | 0.7 | 7.58% | 0.02977 | 0.03763 |
| 0.2 | 0.8 | 8.22% | 0.03695 | 0.04294 |
| 0.1 | 0.9 | 8.86% | 0.04575 | 0.04912 |
| 0 | 1 | 9.50% | 0.05617 | 0.05617 |

11. a.

|  |  |  |
| --- | --- | --- |
| Portfolio | r | σ |
| 1 | 10.0% | 5.1% |
| 2 | 9.0 | 4.6 |
| 3 | 11.0 | 6.4 |

1. See the figure below. The set of portfolios is represented by the curved line. The five points are the three portfolios from Part (a) plus the following two portfolios: one consists of 100% invested in X and the other consists of 100% invested in Y.
2. See the figure below. The best opportunities lie along the straight line. From the diagram, the optimal portfolio of risky assets is portfolio 1, and so Mr. Harrywitz should invest 50 percent in X and 50 percent in Y.



12. a. Expected return = (0.6 × 15) + (0.4 × 20) = 17%

Variance = (0.62 × 202) + (0.42 × 222) + 2(0.6)(0.4)(0.5)(20)(22) = 327.04

Standard deviation = 327.04(1/2) = 18.08%

1. Correlation coefficient = 0 ⇒ Standard deviation = 14.88%

Correlation coefficient = –0.5 ⇒ Standard deviation = 10.76%

1. His portfolio is better. The portfolio has a higher expected return *and* a lower standard deviation.

13. a.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 2003 | 2004 | 2005 | 2006 | 2007 |  | Average | SD | Sharpe Ratio |
| Sauros | 39.1 | 11.0 | 2.6 | 18.0 | 2.3 |  | 14.6 | 15.2 | 0.77 |
| S&P500 | 31.6 | 12.5 | 6.4 | 15.8 | 5.6 |  | 14.4 | 10.5 | 1.09 |
| Risk Free | 1.01 | 1.37 | 3.15 | 4.73 | 4.36 |  | 2.9 | 1.7 |  |

On these numbers she seems to perform worse than the market

b. We can calculate the Beta of her investment as follows (see Table 7.7):

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  | TOTAL |
| Deviation from Average mkt return | 17.2 | -1.9 | -8.0 | 1.4 | -8.8 |  |  |
| Deviation from Average Sauros return | 24.5 | -3.6 | -12.0 | 3.4 | -12.3 |  |  |
| Squared Deviation from Average market return | 296.5 | 3.5 | 63.7 | 2.0 | 77.1 |  | 442.8 |
| Product of Deviations from Average returns | 421.9 | 6.8 | 95.8 | 4.8 | 108.0 |  | 637.2 |
|  |  |  |  |  |  |  |  |
| Market variance | 88.6 |  |  |  |  |  |  |
| Covariance | 127.4 |  |  |  |  |  |  |
| Beta | 1.4 |  |  |  |  |  |  |

To construct a portfolio with a beta of 1.4, we will borrow .4 at the risk free rate and invest this in the market portfolio. This gives us annual returns as follows:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | 2003 | 2004 | 2005 | 2006 | 2007 |  | Average |
| 1.4 times market | 44.2 | 17.5 | 9.0 | 22.1 | 7.8 |  | 20.1 |
| less 0.4 time rf | -0.4 | -0.5 | -1.3 | -1.9 | -1.7 |  | -1.2 |
| net return | 43.84 | 16.95 | 7.70 | 20.23 | 6.10 |  | 19.0 |

The Sauros portfolio does not generate sufficient returns to compensate for its risk.

14. a. The Beta of the first portfolio is 0.714 and offers an average return of 5.9%

|  |  |  |
| --- | --- | --- |
|  | Beta | Expected Return |
| Disney | 0.960 | 7.700 |
| Exxon Mobil | 0.550 | 4.700 |
|  |  |  |
| weighted (40,60) | 0.714 | 5.900 |
|  |  |  |
| Amazon | 2.160 | 22.800 |
| Campbells | 0.300 | 3.100 |
|  |  |  |
| weighted (14.5, 85.5) | 0.570 | 5.858 |

b. We can devise a superior portfolio with a blend of 85.5% Campbell’s Soup and 14.5% Amazon.

c.

|  |  |  |
| --- | --- | --- |
|  | Beta | Expected Return |
| Amazon | 2.160 | 22.800 |
| Dell | 1.410 | 13.400 |
|  |  |  |
| weighted (40,60) | 1.710 | 17.160 |
|  |  |  |
| Ford | 1.750 | 19.000 |
| Campbell Soup | 0.300 | 3.100 |
|  |  |  |
| weighted (89, 11) | 1.591 | 17.251 |
|  |  |  |

15. a.

b. Market risk premium = rm – rf = 0.12 – 0.04 = 0.08 = 8.0%

1. Use the security market line:

r = rf + β(rm – rf)

r = 0.04 + [1.5 × (0.12 – 0.04)] = 0.16 = 16.0%

d. For any investment, we can find the opportunity cost of capital using the security market line. With β = 0.8, the opportunity cost of capital is:

r = rf + β(rm – rf)

r = 0.04 + [0.8 × (0.12 – 0.04)] = 0.104 = 10.4%

The opportunity cost of capital is 10.4% and the investment is expected to earn 9.8%. Therefore, the investment has a negative NPV.

e. Again, we use the security market line:

r = rf + β(rm – rf)

0.112 = 0.04 + β(0.12 – 0.04) ⇒ β = 0.9

16. a. Percival’s current portfolio provides an expected return of 9% with an annual standard deviation of 10%. First we find the portfolio weights for a combination of Treasury bills (security 1: standard deviation = 0%) and the index fund (security 2: standard deviation = 16%) such that portfolio standard deviation is 10%. In general, for a two security portfolio:

σP2 = x12σ12 + 2x1x2σ1σ2ρ12 + x22σ22

(0.10)2 = 0 + 0 + x22(0.16)2

x2 = 0.625 ⇒ x1 = 0.375

Further:

rp = x1r1 + x2r2

rp = (0.375 × 0.06) + (0.625 × 0.14) = 0.11 = 11.0%

Therefore, he can improve his expected rate of return without changing the risk of his portfolio.

1. With equal amounts in the corporate bond portfolio (security 1) and the index fund (security 2), the expected return is:

rp = x1r1 + x2r2

rp = (0.5 × 0.09) + (0.5 × 0.14) = 0.115 = 11.5%

σP2 = x12σ12 + 2x1x2σ1σ2ρ12 + x22σ22

σP2 = (0.5)2(0.10)2 + 2(0.5)(0.5)(0.10)(0.16)(0.10) + (0.5)2(0.16)2

σP2 = 0.0097

σP = 0.985 = 9.85%

Therefore, he can do even better by investing equal amounts in the corporate bond portfolio and the index fund. His expected return increases to 11.5% and the standard deviation of his portfolio decreases to 9.85%.

17. First calculate the required rate of return (assuming the expansion assets bear the same level of risk as historical assets):

r = rf + β(rm – rf)

r = 0.04 + [1.4 × (0.12 – 0.04)] = 0.152 = 15.2%

The use this to discount future cash flows; NPV = -25.29

|  |  |  |  |
| --- | --- | --- | --- |
| Year | Cash Flow | Discount factor | PV |
| 0 | -100 | 1 | -100.00 |
| 1 | 15 | 0.868 | 13.02 |
| 2 | 15 | 0.754 | 11.30 |
| 3 | 15 | 0.654 | 9.81 |
| 4 | 15 | 0.568 | 8.52 |
| 5 | 15 | 0.493 | 7.39 |
| 6 | 15 | 0.428 | 6.42 |
| 7 | 15 | 0.371 | 5.57 |
| 8 | 15 | 0.322 | 4.84 |
| 9 | 15 | 0.280 | 4.20 |
| 10 | 15 | 0.243 | 3.64 |
|  |  |  |  |
| NPV |  |  | -25.29 |

18. a. True. By definition, the factors represent macro-economic risks that cannot be eliminated by diversification.

1. False. The APT does not specify the factors.
2. True. Different researchers have proposed and empirically investigated different factors, but there is no widely accepted theory as to what these factors should be.
   1. True. To be useful, we must be able to estimate the relevant parameters. If this is impossible, for whatever reason, the model itself will be of theoretical interest only.

19. Stock P: r = 5% + (1.0 × 6.4%) + [(–2.0) × (–0.6%)] + [(–0.2) × 5.1%] = 11.58%

Stock P2: r = 5% + (1.2 × 6.4%) + [0 × (–0.6%)] + (0.3 × 5.1%) = 14.21%

Stock P3: r = 5% + (0.3 × 6.4%) + [0.5 × (–0.6%)] + (1.0 × 5.1%) = 11.72%

20. a. Factor risk exposures:

b1(Market) = [(1/3)×1.0] + [(1/3)×1.2] + [(1/3)×0.3] = 0.83

b2(Interest rate) = [(1/3)×(–2.0)] +[(1/3)×0] + [(1/3)×0.5] = –0.50

b3(Yield spread) = [(1/3)×(–0.2)] + [(1/3)×0.3] + [(1/3)×1.0] = 0.37

1. rP = 5% + (0.83×6.4%) + [(–0.50)×(–0.6%)] + [0.37×5.1%] = 12.50%

21. rBoeing = 0.2% + (0.66 × 7%) + (01.19 × 3.6%) + (-0.76 × 5.2%) = 5.152%

RJ&J = 0.2% + (0.54 × 7%) + (-0.58 × 3.6%) + (0.19 × 5.2%) = 2.88%

RDow = 0.2% + (1.05 × 7%) + (–0.15 × 3.6%) + (0.77 × 5.2%) = 11.014%

rMsft= 0.2% + (0.91 × 7%) + (0.04 × 3.6%) + (–0.4 × 5.2%) = 4.346%

22. In general, for a two-security portfolio:

σp2 = x12σ12 + 2x1x2σ1σ2ρ12 + x22σ22

and:

x1 + x2 = 1

Substituting for x2 in terms of x1 and rearranging:

σp2 = σ12x12 + 2σ1σ2ρ12(x1 – x12) + σ22(1 – x1)2

Taking the derivative of σp2 with respect to x1, setting the derivative equal to zero and rearranging:

x1(σ12 – 2σ1σ2ρ12 + σ22) + (σ1σ2ρ12 – σ22) = 0

Let Campbell Soup be security one (σ1 = 0.158) and Boeing be security two (σ2 = 0.237). Substituting these numbers, along with ρ12 = 0.18, we have:

x1 = 0.731

Therefore:

x2 = 0.269

23. a. The ratio (expected risk premium/standard deviation) for each of the four portfolios is as follows:

Portfolio A: (22.8 – 10.0)/50.9 = 0.251

Portfolio B: (10.5 – 10.0)/16.0 = 0.031

Portfolio C: (4.2 – 10.0)/8.8 = -0.659

Therefore, an investor should hold Portfolio A.

1. The beta for Amazon relative to Portfolio A is identical.

|  |  |
| --- | --- |
|  |  |

1. If the interest rate is 5%, then Portfolio C becomes the optimal portfolio, as indicated by the following calculations:

Portfolio A: (22.8 – 5.0)/50.9 = 0.35

Portfolio B: (10.5 – 5.0)/16.0 = 0.344

Portfolio C: (4.2 – 5.0)/8.8 = -0.091

The results do not change.

24. Let rx be the risk premium on investment X, let xx be the portfolio weight of X (and similarly for Investments Y and Z, respectively).

a. rx = (1.75×0.04) + (0.25×0.08) = 0.09 = 9.0%

ry = [(–1.00)×0.04] + (2.00×0.08) = 0.12 = 12.0%

rz = (2.00×0.04) + (1.00×0.08) = 0.16 = 16.0%

1. This portfolio has the following portfolio weights:

xx = 200/(200 + 50 – 150) = 2.0

xy = 50/(200 + 50 – 150) = 0.5

xz = –150/(200 + 50 – 150) = –1.5

The portfolio’s sensitivities to the factors are:

Factor 1: (2.0×1.75) + [0.5×(–1.00)] – (1.5×2.00) = 0

Factor 2: (2.0×0.25) + (0.5×2.00) – (1.5×1.00) = 0

Because the sensitivities are both zero, the expected risk premium is zero.

1. This portfolio has the following portfolio weights:

xx = 80/(80 + 60 – 40) = 0.8

xy = 60/(80 + 60 – 40) = 0.6

xz = –40/(80 + 60 – 40) = –0.4

The sensitivities of this portfolio to the factors are:

Factor 1: (0.8×1.75) + [0.6×(–1.00)] – (0.4×2.00) = 0

Factor 2: (0.8×0.25) + (0.6×2.00) – (0.4×1.00) = 1.0

The expected risk premium for this portfolio is equal to the expected risk premium for the second factor, or 8 percent.

1. This portfolio has the following portfolio weights:

xx = 160/(160 + 20 – 80) = 1.6

xy = 20/(160 + 20 – 80 ) = 0.2

xz = –80/(160 + 20 – 80) = –0.8

The sensitivities of this portfolio to the factors are:

Factor 1: (1.6×1.75) + [0.2×(–1.00)] – (0.8×2.00) = 1.0

Factor 2: (1.6×0.25) + (0.2×2.00) – (0.8×1.00) = 0

The expected risk premium for this portfolio is equal to the expected risk premium for the first factor, or 4 percent.

1. The sensitivity requirement can be expressed as:

Factor 1: (xx)(1.75) + (xy)(–1.00) + (xz)(2.00) = 0.5

In addition, we know that:

xx + xy + xz = 1

With two linear equations in three variables, there is an infinite number of solutions. Two of these are:

1. xx = 0 xy = 0.5 xz = 0.5

2. xx = 6/11 xy = 5/11 xz = 0

The risk premiums for these two funds are:

r1 = 0×[(1.75 × 0.04) + (0.25 × 0.08)]

+ (0.5)×[(–1.00 × 0.04) + (2.00 × 0.08)]

+ (0.5)×[(2.00 × 0.04) + (1.00 × 0.08)] = 0.14 = 14.0%

r2 = (6/11)×[(1.75 × 0.04) + (0.25 × 0.08)]

+(5/11)×[(–1.00 × 0.04) + (2.00 × 0.08)]

+0 × [(2.00 × 0.04) + (1.00 × 0.08)] = 0.104 = 10.4%

These risk premiums differ because, while each fund has a sensitivity of 0.5 to factor 1, they differ in their sensitivities to factor 2.

1. Because the sensitivities to the two factors are the same as in Part (b), one portfolio with zero sensitivity to each factor is given by:

xx = 2.0 xy = 0.5 xz = –1.5

The risk premium for this portfolio is:

(2.0×0.08) + (0.5×0.14) – (1.5×0.16) = –0.01

Because this is an example of a portfolio with zero sensitivity to each factor and a nonzero risk premium, it is clear that the Arbitrage Pricing Theory does not hold in this case.

A portfolio with a positive risk premium is:

xx = –2.0 xy = –0.5 xz = 1.5